Fast FDTD Simulation Using Laguerre Polynomials in MNA Framework

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Abstract— **In this paper, a fast transient simulation scheme using Laguerre polynomials, called Laguerre-MNA, has been developed for FDTD and circuit simulation. Companion models for the Yee cells and circuit components have been derived, permitting the use of MNA analysis to perform FDTD/transient circuit simulation using the Laguerre method. Companion models help simplify the matrix setup and reduce the matrix dimension that needs to be solved without employing long cumbersome equations. FDTD simulation using Laguerre polynomials is unconditionally stable and has shown to be much faster than conventional FDTD scheme. Prior work on transient electromagnetic simulation using Laguerre polynomials has a drawback of being able to simulate only for a certain time-duration. A memory/time efficient solution has been proposed by which simulation can be done for all time.**

I. Introduction

Time-domain techniques, such as the finite-difference time-domain (FDTD) method, Latency Insertion Method (LIM) [1] and Spice transient simulation, have been widely used to analyze power-integrity (PI), signal-integrity (SI) and electromagnetic interference (EMI) problems. The limitation of FDTD and LIM is the Courant-Friedrich-Levy stability condition that limits the maximum time-step that can be used, in order to obtain stable simulation results [2]. An unconditionally stable transient EM simulation method using Laguerre polynomials was proposed in [3]. The method proposed in [3] for electromagnetic simulation has shown to be $80\times$ faster than conventional FDTD scheme.

In this paper, companion models for the FDTD grid and linear circuit components have been proposed, so that modified nodal analysis (MNA) can be used to carry out FDTD/transient circuit simulation. The advantages of this are (1) Laguerre-MNA can be integrated seamlessly with Spice to do FDTD/transient circuit simulation, and (2) circuit representation simplifies the matrix setup and helps reduce the matrix dimension to be solved without long cumbersome equations. The requirement for using LIM is that, all branches should have an inductor and all nodes must have a capacitor connected to ground, in order carry out the time-stepping. However, in the Laguerre-MNA method for circuit simulation, there are no such conditions and can be applied to any arbitrary linear network, comparable in performance to Spice.

Reference [3] has the drawback of being able to simulate only for a certain time-duration. As a consequence of this limitation, [3] can only be applied to structures where the fields decay to zero within the time-duration for which simulation can be carried out. Typically, in the analysis of SI/PI/EMI, resonant structures can cause the waveforms to decay very slowly and simulation needs to be carried out longer than the time-duration supported by [3]. A time/memory efficient solution has been proposed, so that simulation can be done for all time, as well as all structures.

The remaining sections are organized as follows: transient simulation methodology is presented in Sec. II; each of the steps in the method is explained in more detail in Sec. III-VII; limitations from prior work and solution to overcome this limitation is given in Sec. IV and a summary of this paper in Sec. VIII.

II. Transient Simulation Methodology

Transient FDTD/circuit simulation using Laguerre polynomials is presented in this section. Laguerre-MNA can be used for (1) EM analysis by FDTD, and (2) transient simulation in circuits composed of resistors, capacitors, inductors, voltage/current sources. The flowchart for simulation is shown in Fig. 1. The first step is to represent the source waveforms in time-domain into equivalent representations in the Laguerre-domain. The time-domain waveforms are represented as a sum of Laguerre-polynomials that are scaled by Laguerre basis coefficients. This representation is explained in Section III. The second step is to replace (1) the FDTD grid, or in the case of circuit simulation, (2) capacitors and inductors with their equivalent Laguerre-domain companion models. The circuit models are given in Sec. V. The transient sources are replaced with DC sources. For each of the values in the Laguerre-domain that represents the time-domain source waveform, a DC analysis is done once. The solution at the end of each DC analysis is used to update the companion models, before the next DC analysis is performed. After updating the companion models, a DC analysis is carried out using the next value in the Laguerre-domain that represents the source waveform. These series of steps are given by Steps 3-5 in Fig. 1. The final step is to construct the time-domain waveform for the output of interest from the DC solutions.

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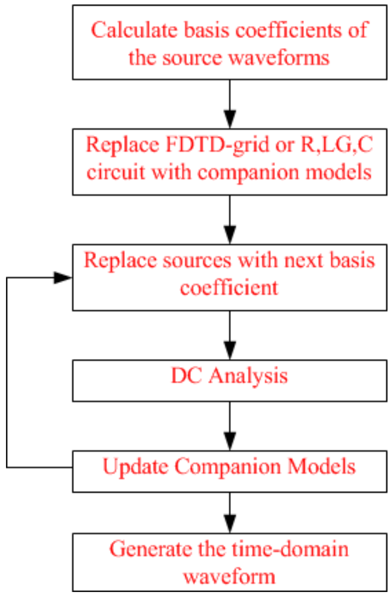


Fig. 1. Flowchart for transient simulation using Laguerre polynomials

III. Step 1: Laguerre Basis Functions

The first step is to represent the time-domain source waveforms into equivalent Laguerre-domain representations. A transient source waveform $W(t)$ can be represented as a sum of N Laguerre basis functions $\varphi_p(\bar{t})$, scaled by Laguerre basis coefficients W_p as shown in Eq. 1 [3].

$$W(t) = \sum_{p=0}^{p=N-1} W_p \varphi_p(\bar{t}) \tag{1}$$

$$\bar{t} = s \cdot t \tag{2}$$

In Eq. 2, $\bar{t}$ is the real time t multiplied by a scaling factor s. The actual time scale at which the simulation is run is very small, typically picoseconds when rise/fall times are in the order of picoseconds. To make the basis function work, the real time is multiplied by s to scale the magnitude in the order of seconds. The basis functions for orders $p = 0 - 4$ are plotted in Figure 2 [3]. The basis functions span a time in the order of seconds as shown by the x-axis in Figure 2, hence the need for the scale factor.

The definition of the basis function is given by Eq. 3.

$$\varphi_u(\bar{t}) = e^{-\bar{t}/2} L_u(\bar{t}) \tag{3}$$

Laguerre polynomials are defined recursively as follows:

$$L_0(\bar{t}) = 1 \tag{4}$$

$$L_1(\bar{t}) = 1 - \bar{t} \tag{5}$$

$$pL_p(\bar{t}) = (2p - 1 - \bar{t})L_{p-1}(\bar{t}) - (p-1)L_{p-2}(\bar{t}), \quad for\ p \geq 2 \tag{6}$$

The transient source waveforms are replaced by DC sources. The values of the DC sources are the set of Laguerre basis coefficients $\{W_p\}$, which represent the transient source waveform in the Laguerre-domain. $\{W_p\}$ is generated from $W(t)$ using Eq. 7.

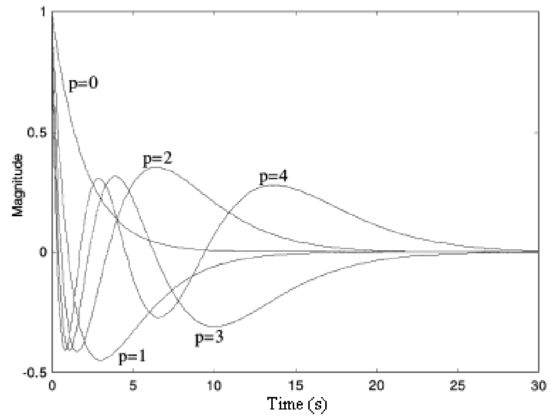


Fig. 2. Laguerre basis functions for order p=0-4

$$W_p = \int_0^{\infty} W(t)\varphi_p(\bar{t})d\bar{t} \tag{7}$$

Laguerre basis functions satisfy the orthonormal property,

$$\int_0^{\infty} \varphi_u(\bar{t})\varphi_v(\bar{t})d\bar{t} = \delta_{uv} \tag{8}$$

In Eq. 8, δ_{uv} is the Kronecker delta function. Equation 7 can be derived by multiplying both sides of Eq. 1 with $\varphi_q(\bar{t})$, integrating the two sides from $[0, \infty]$ and using the orthonormal property given by Eq. 8.

The output of Step 1 is to compute the set of Laguerre basis coefficients $\{W_p\}$ for each of the transient source waveforms.

IV. Limitations in Prior Work and Solution

Transient simulation using Laguerre polynomials was originally used in electromagnetic simulation. A drawback of the methodology in [3] is that the transient simulation can be performed only for a certain time-duration and cannot be done for all time. There are two reasons for this limitation: The first reason is due to the nature of the Laguerre basis functions and the second reason is due to the finite precision of the computer, which makes it impossible to represent very large numbers or very small numbers.

The first reason for the limitation is due to the nature of the basis functions. The Laguerre basis functions for $p = 0 - 4$ are plotted in Figure 2. As shown in the figure, all of the basis functions approach 0, as t tends to ∞. Therefore, any time-domain waveform that is spanned by these set of basis functions also goes to 0 as t tends to ∞. Circuits that are lossless or have a low loss cannot be simulated accurately, because the waveforms can be non-zero for a long period of time.

The second reason for the limitation is due to the finite precision of the computer. The Laguerre basis function is an exponentially decaying function multiplied by a Laguerre polynomial, as given by Eq. 3. The exponential function quickly decays to 0, and beyond a certain time-point the exponential function is approximated with a 0. Laguerre polynomials become very large with increasing time. Beyond a certain time, the numbers become very large to be represented with the limitation of finite precision and is represented as *Inf* in the *IEEE 754* floating point standard. Consequently, beyond a certain time-point, the basis function is represented as $0 \times Inf$ or *NaN*, not a number.

The solution to overcome this limitation is to divide the simulation time into different intervals. Let Interval *I* span from time $t = t_0$ to $t = t_1$, Interval *II* span from time $t = t_1$ to $t = t_2$, and so on. The length of each interval is chosen such that, simulation can be accurately performed in that time duration. The final values at the end of Interval *I* are used as initial conditions to simulate in Interval *II*. This process is repeated until the time duration for which the simulation needs to be done is completed.

The differential equations have been modified to include initial conditions, before converting them into Laguerre domain. The companion models presented in this paper include initial conditions. Using the proposed solution, simulation can done for all time duration.

V. Companion Models for FDTD-grid and Circuit Components

A. FDTD-grid 1D

Consider a 1D FDTD grid shown in Fig. 3. The fields present are H_y and E_z, excited by J_z current source. The positions of the electric fields are marked by $|$ and those of the magnetic fields are shown by $\times$. The boundary conditions on either side of the grid are perfect electric conductor (PEC) boundary conditions.

The companion model of the FDTD grid is described before the derivation. The circuit model of a unit-cell in an FDTD grid in terms of resistors and voltage sources are given by the second subfigure in Fig. 3. At the end of q^{th} DC analysis, the nodal voltages and branch currents represent the q^{th} Laguerre basis coefficients of the electric fields and the magnetic fields, respectively. The value of the q^{th} Laguerre basis coefficient of the electric field $E_z|_i^q$ is represented by the nodal voltage marked V_i^q and the magnetic fields on either side of $E_z|_i^q$, $H_y|_{i-1/2}^q$ and $H_y^q|_{i+1/2}$, are given by the branch currents $I_{i-1/2}^q$ and $I_{i+1/2}^q$, respectively. The circuit model of the unit cell is cascaded to represent as many unit cells as needed. The model is terminated by a short-circuit on both the sides to represent the perfect electric conductor boundary conditions. The values of the components are,

$$R_1 = \frac{s\mu\Delta x}{2} \tag{9}$$

$$I_{val,i-1/2}^q = 2H_y^{init}|_{i-1/2} - 2\Bigg(\sum_{k=0,q>0}^{q-1} H_y^k|_{i-1/2}\Bigg) \tag{10}$$

$$R_{TH} = \frac{2}{s\epsilon\Delta x} \tag{11}$$

$$V_{TH}^q = \frac{-2J_z^q|_i}{s\epsilon} - 2\Bigg(\sum_{k=0,q>0}^{q-1} E_z^k|i\Bigg) + 2E_z^{init}|_i \tag{12}$$

μ, ϵ represent the material properties of the medium; Δx is the unit-cell dimension; H_y^{init} and E_z^{init} are the initial conditions of the electric and magnetic fields at the location marked by their subscripts; s is the time-scale factor given by Eq. 2.

The remaining section presents the derivation of the circuit model. Maxwell's equations with initial conditions in 1D can be written as,

$$\frac{\partial H_y}{\partial t} - H_y(\vec{r},0)\delta(t) = \frac{1}{\mu}\frac{\partial E_z}{\partial x} \tag{13}$$

$$\frac{\partial E_z}{\partial t} - E_z(\vec{r},0)\delta(t) = \frac{1}{\epsilon}\left[\frac{\partial H_y}{\partial x} - J_z\right] \tag{14}$$

$H_y(\vec{r},0)$ and $E_z(\vec{r},0)$ are the initial values of the magnetic and electric fields at position $\vec{r}$, and beginning of a time-interval; $\delta(t)$ is the Dirac delta function. $H_y(\vec{r},t), E_z(\vec{r},t)$ and $J_z(\vec{r},t)$ can be written as a sum of N Laguerre basis coefficients given by,

$$H_y(\vec{r},t) = \sum_{q=0}^{N-1} H_y^q(\vec{r})\varphi_q(\bar{t}) \tag{15}$$

$$E_z(\vec{r},t) = \sum_{q=0}^{N-1} E_z^q(\vec{r})\varphi_q(\bar{t}) \tag{16}$$

$$J_z(\vec{r},t) = \sum_{q=0}^{N-1} J_z^q(\vec{r})\varphi_q(\bar{t}) \tag{17}$$

Substituting Eq. 15-17 into Eq. 13-14, using the time-derivative relationship given in [3], and by applying the orthonormal property of the Laguerre basis functions given in Eq. 8, the following equations can be obtained:

$$\begin{aligned} H_y^q|_{i+1/2} &= -2\Bigg(\sum_{k=0,q>0}^{q-1} H_y^k|_{i+1/2}\Bigg) + \\ & 2H_y^{init}|_{i+1/2} + \frac{2}{s\mu\Delta x}\Bigg(E_z^q|_{i+1} - E_z^q|_i\Bigg) \end{aligned} \tag{18}$$

$$\begin{aligned} E_z^q|_i &= -2\Bigg(\sum_{k=0,q>0}^{q-1} E_z^k|_i\Bigg) + 2E_z^{init}|_i + \\ & \frac{2}{s\epsilon\Delta x}\Bigg[H_y^q|_{i+1/2} - H_y^q|_{i-1/2}\Bigg] - \frac{2}{s\epsilon}J_z^q|_i \end{aligned} \tag{19}$$

In deriving Eq. 18-19, Eq. 20 is used when integrating the delta function term.

$$\int_0^\infty \delta(t)\varphi_p(\bar{t})d\bar{t} = s\varphi_p(0) = s \tag{20}$$

The circuit model of the FDTD grid in Fig. 3 satisfies Eq.18-19. The PEC boundary condition dictates that the electric field be zero for all the Laguerre coefficients. This is taken care of by a *short* circuit, forcing the electric field Laguerre coefficients to be 0.

The number of unknowns to be solved in MNA analysis are the unknown nodal voltages and the unknown currents through the voltage sources V_{TH}^q. One possible way of reducing the matrix dimension that needs to be solved is by substituting Eq. 18 into Eq. 19, so that the unknowns to be solved

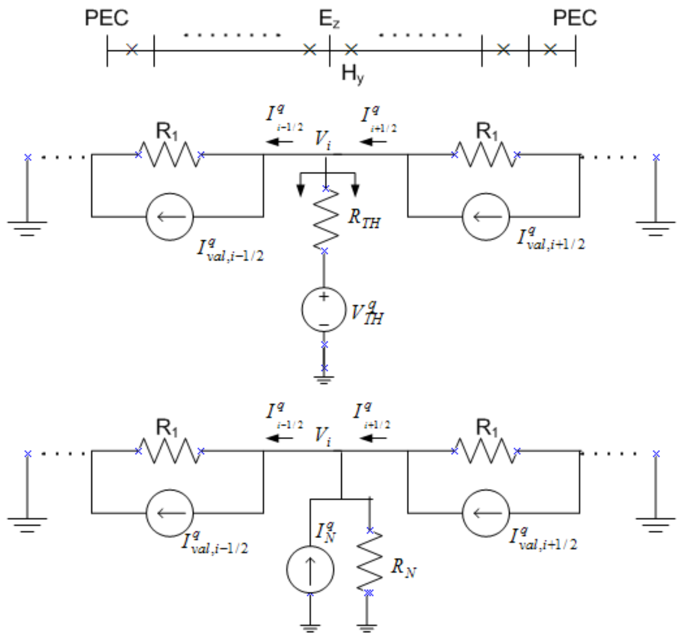


Fig. 3. Companion model for a unit cell in an FDTD grid

are only the coefficients of the electric field; other terms involving coefficients of the magnetic field represent *history terms* that have been solved in previous analyses. However, this procedure is very cumbersome due to the length of the equations that needs to be manipulated. An easier approach is to convert the Thevenin representation of the circuit, looking into the circuit marked by the double arrow, into a Norton form as given by the third subfigure in Fig. 3:

$$R_N = R_{TH} \tag{21}$$

$$I_N^q = \frac{V_{TH}^q}{R_{TH}} \tag{22}$$

In the Norton representation, the unknowns are only the nodal voltages (electric field coefficients). The branch currents that represent the magnetic field Laguerre coefficients can be obtained by applying Ohm's law using the solved nodal voltages.

The companion model is updated using the q^{th} DC solution, before performing the $q+1$ DC analysis.

B. FDTD-grid 2D

Consider a 2D FDTD grid with H_z, E_x, E_y fields and J_y source. The field arrangement is shown in Fig. 4(a). Time-domain Maxwell's differential equations without including the initial conditions are given in [3]. Including initial conditions, similar to the 1D case, the Laguerre representation of Maxwell's equations for the 2D case, consists of the following set:

$$E_y^q|_{i,j+\frac{1}{2}} - 2E_y^{init}|_{i,j+\frac{1}{2}} = -C_x^E|_{i,j}\left(H_z^q|_{i+\frac{1}{2},j+\frac{1}{2}} - H_z^q|_{i-\frac{1}{2},j+\frac{1}{2}}\right) - \frac{2}{s\epsilon}J_y^q|_{i,j+\frac{1}{2}} - 2\sum_{k=0,q>0}^{q-1} E_y^k|_{i,j+\frac{1}{2}} \tag{23}$$

$$E_x^q|_{i+\frac{1}{2},j} - 2E_x^{init}|_{i+\frac{1}{2},j} = C_y^E|_{i,j}\left(H_z^q|_{i+\frac{1}{2},j+\frac{1}{2}} - H_z^q|_{i+\frac{1}{2},j-\frac{1}{2}}\right) - 2\sum_{k=0,q>0}^{q-1} E_x^k|_{i+\frac{1}{2},j} \tag{24}$$

$$H_z^q|_{i+\frac{1}{2},j+\frac{1}{2}} - 2H_z^{init}|_{i+\frac{1}{2},j+\frac{1}{2}} = -C_x^H|_{i,j}\left(E_y^q|_{i+1,j+\frac{1}{2}} - E_y^q|_{i,j+\frac{1}{2}}\right) + C_y^H|_{i,j}\left(E_x^q|_{i+\frac{1}{2},j+1} - E_x^q|_{i+\frac{1}{2},j}\right) - 2\sum_{k=0,q>0}^{q-1} H_z^k|_{i+\frac{1}{2},j+\frac{1}{2}} \tag{25}$$

where,

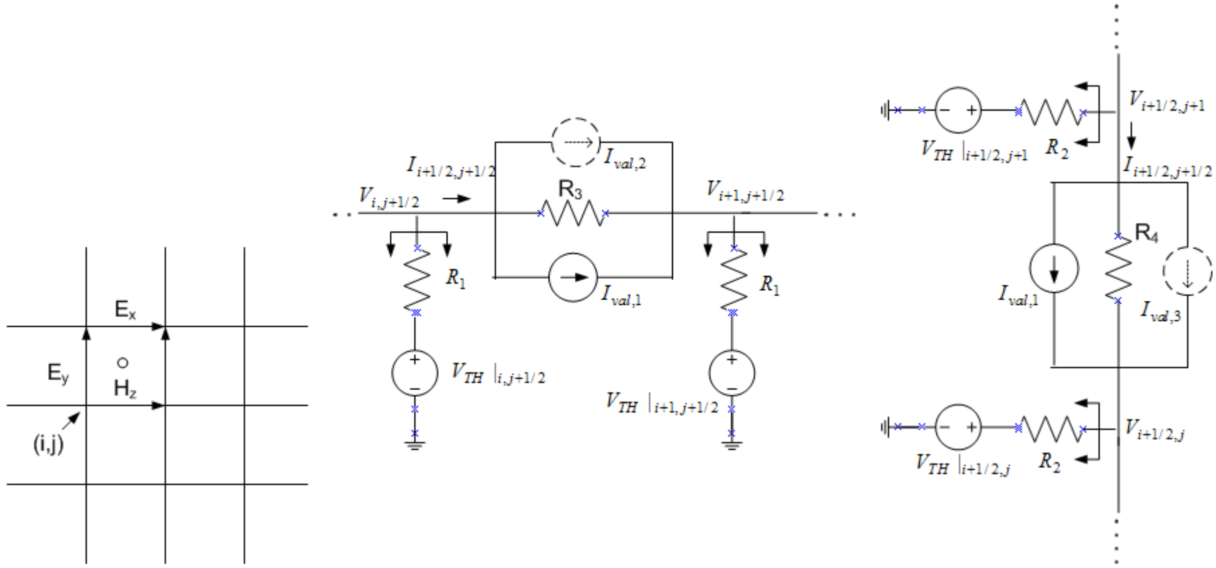
$$C_y^E|_{i,j} = \frac{2}{s\epsilon_{i,j}\Delta y_j}; C_x^E|_{i,j} = \frac{2}{s\epsilon_{i,j}\Delta x_i} \tag{26}$$

$$C_x^H|_{i,j} = \frac{2}{s\mu_{i,j}\Delta x_i}; C_y^H|_{i,j} = \frac{2}{s\mu_{i,j}\Delta y_j} \tag{27}$$

The two subfigures in Fig. 4(b) represent the circuit model of the unit-cell shown in Fig. 4(a). The nodal voltages $V_{i,j+1/2}$ and $V_{i+1,j+1/2}$ in the first subfigure in Fig. 4(b) represent q^{th} Laguerre electric field basis coefficients, $E_y^q|_{i,j+1/2}$ and $E_y^q|_{i+1,j+1/2}$, respectively. The nodal voltages $V_{i+1/2,j}$ and $V_{i+1/2,j+1}$ in the second subfigure in Fig. 4(b) are q^{th} Laguerre electric field coefficients $E_x^q|_{i+1/2,j}$ and $E_x^q|_{i+1/2,j+1}$, respectively. The branch current marked $I_{i+1/2,j+1/2}$ are the same values in both the subfigures and represent the q^{th} Laguerre magnetic field coefficients $H_z^q|_{i+1/2,j+1/2}$.
The values of R_1 and $V_{TH}|_{i,j+1/2}$ are,

$$R_1 = C_x^E \tag{28}$$

$$V_{TH}|_{i,j+1/2} = 2E_y^{init}|_{i,j+\frac{1}{2}} - \frac{2}{s\epsilon}J_y^q|_{i,j+\frac{1}{2}}$$



(a) 2D FDTD Grid

(b) Circuit model for the 2D FDTD grid

Fig. 4. Simulation results

$$-2\sum_{k=0,q>0}^{q-1} E_y^k|_{i,j+\frac{1}{2}} \tag{29}$$

The values of R_2 and $V_{TH}|_{i+1/2,j}$ are,

$$R_2 = C_y^E \tag{30}$$

$$V_{TH}|_{i+1/2,j} = 2E_x^{init}|_{i+\frac{1}{2},j} - 2\sum_{k=0,q>0}^{q-1} E_x^k|_{i+\frac{1}{2},j} \tag{31}$$

Eq. 28-29 model Eq. 23; Eq. 30-31 model Eq. 24.
The values of $I_{val,1}$, $I_{val,2}$, $I_{val,3}$, R_3 and R_4 are,

$$I_{val,1} = 2H_z^{init}|_{i+\frac{1}{2},j+\frac{1}{2}} - 2\sum_{k=0,q>0}^{q-1} H_z^k|_{i+\frac{1}{2},j+\frac{1}{2}} \tag{32}$$

$$I_{val,2} = C_y^H\Big(V_{i+1/2,j+1} - V_{i+1/2,j}\Big) \tag{33}$$

$$I_{val,3} = C_x^H\Big(V_{i,j+1/2} - V_{i+1,j+1/2}\Big) \tag{34}$$

$$R_3 = \frac{1}{C_x^H}; R_4 = \frac{1}{C_y^H} \tag{35}$$

Eq. 32-35 model Eq. 25. $I_{val,2}$ and $I_{val,3}$ in Fig. 4(b) are shown by dotted circles and are voltage controlled current sources, that couple the two circuits together. It can be seen from KCL and KVL equations that Eq. 28-35 represent Eq. 23-25.

The number of unknowns that needs to be solved using MNA analysis can be reduced by converting Thevenin representations into Norton equivalents. Looking into the circuit marked by the *double arrow* shown in Fig. 4(b), Thevenin circuit can be converted into a Norton model, as explained in Sec. V-A. The number of unknowns can also be reduced by substituting Eq. 25 into Eq. 23-24, such that only the electric field Laguerre coefficients needs to be solved. However, this is a lot more cumbersome than converting from Thevenin to Norton equivalent circuit form. It should be noted that both of these methods to reduce the number of unknowns, result in the same matrix dimension. However, reducing the unknowns by Thevenin to Norton conversion is much simpler.

C. *Circuit Components: R,L,G,C elements*

Laguerre-domain companion models for inductors, capacitors are given in the work done by the authors of this paper in [4]. Resistors, conductances in Laguerre-MNA are treated in the same fashion as conventional MNA analysis. Cases for which Laguerre-MNA can be faster than MNA analysis are also given in [4].

VI. Steps 3-5: DC Analysis

This subsection explains Steps 3-5 in the flowchart shown in Fig. 1. The FDTD grid/R,L,G,C components are replaced with their companion models. The transient sources are replaced by DC sources. Assuming that the sources are represented by N Laguerre basis coefficients, as given by Equation 1, N DC analysis are performed. In the $m^{th}, m \le N$ DC analysis step, the value of the m^{th} Laguerre basis coefficient is used for the DC source. At the end of the DC analysis (Step 4), the solution from the DC analysis are used to update the companion models (Step 5). Steps 3-5 are repeated for the $(m+1)^{th}$ Laguerre basis coefficient as the DC source value, doing the DC analysis and updating the companion models at the end of the DC analysis. This process is repeated until N DC analyses have been done.

The DC solution of the output of interest at the end of the m^{th} DC analysis step, represents the m^{th} Laguerre basis coefficient of the corresponding time-domain waveform.

It must be noted that Laguerre-MNA does not require storing all of the solution from the series of DC analysis that has been performed. At the end of each DC analysis, once the companion models have been updated, there is no need for saving the solution. The only solution that needs to be stored at the end of each DC analysis is the solution of the output for which the transient waveform is to be observed.

The final time-domain values at the end of an interval, e.g. Interval Q, must be computed in order to use these values as the initial conditions in the next time-interval, Interval (Q+1). Not all the coefficients, i.e. the DC solution, need to be saved, in order to compute the final value at the end of a time-interval. At the end of each DC analysis, the contribution of p^{th} Laguerre basis coefficient (W_p) to the final value of the transient waveform at the end of a time-interval (t_f) can be computed by using Eq. 36.

$$value(t_f) = value(t_f) + W_p \varphi_p(st_f) \tag{36}$$

$value(t_f)$ is first initialized to 0, before using Eq. 36.

VII. Step 6: Time-domain Waveform

The final step is to obtain the time-domain waveform, from all the DC solution of the output quantity. The time-domain waveform can be obtained by using Eq. 1. The p^{th} basis coefficient is multiplied by the p^{th} basis function; assuming there are N basis coefficients that represent the output waveform, the N waveforms are added to obtain the output transient waveform.

Consider the L,C network shown in Fig. 5 with 102 nodes. The test-case is similar in structure to a model of the parasitics of a power/ground plane for power-integrity analysis. The values of the capacitors and inductors are 1nF and 1nH, except for the last three marked in red, which are 1fF and 1fH. In Circuit-FDTD, the presence of the small valued capacitors and inductors, considerably reduces the time-step due to the Courant condition. Laguerre-MNA, which is unconditionally stable, has a significant advantage over Circuit-FDTD. Simulation using Circuit-FDTD for 100ns will require over 1.0×10^8 iterations, but simulation using Laguerre-MNA for this example requires only an iteration 121 times to solve for 121 basis coefficients.

Voltage waveform at node 102 are plotted in Fig. 6. The dotted blue curve is obtained by using WinSpice and the solid red curve is by using Laguerre-MNA. The input waveform is a triangular input with a rise/fall time of 4ns, and a delay of 1ns; the voltage source is connected to node 1; the time-scale factor used is $s = 3.8 \times 10^9$. There is a good correlation between WinSpice and Laguerre-MNA.

The same structure of the companion models between Laguerre-MNA and MNA makes it possible to simulate even very large sized problems using Laguerre-MNA rather than MNA [4] [5].

VIII. Conclusions

In this paper, a fast transient simulation method called Laguerre-MNA has been proposed by which FDTD simulation/linear transient circuit simulation can be done with

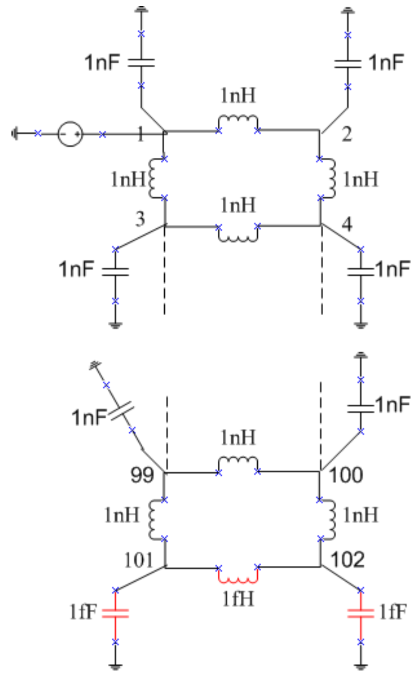


Fig. 5. An L,C network

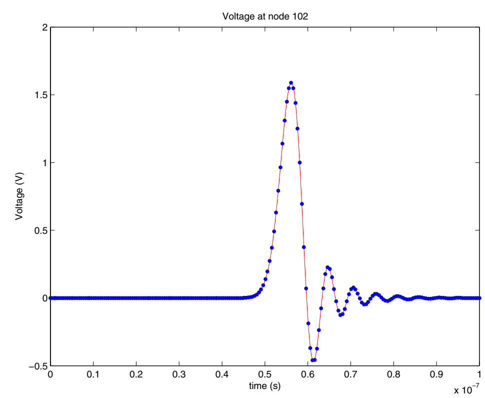


Fig. 6. Voltage at node 102; dots: WinSpice and solid: Laguerre-MNA

Laguerre polynomials, using MNA analysis. Stamp rule can be used to setup the MNA matrix, simplifying the matrix setup. MNA analysis can reduce the matrix dimension to be solved significantly, without using long cumbersome equations.

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