Wideband Electrical Modeling of Large Three-Dimensional Interconnects using Accelerated Generation of Partial Impedances with Cylindrical Conduction Mode Basis Functions

Ki Jin Han, Madhavan Swaminathan, and Ege Engin

School of Electrical and Computer Engineering, Georgia Institute of technology, 266 Ferst Drive, Atlanta, Georgia 30332, U.S.A.

Abstract **— For wideband modeling of large and complicated three-dimensional interconnects, this paper proposes an efficiency improvement in solving electric field integral equation with cylindrical conduction mode basis functions. Based on the multifunction method, the improved method reduces computational cost by using smaller number of higher-order basis functions for computing mutual inductances between far-separated conductors. From the modeling examples of through-hole vias and bonding wires in stacked IC's, the proposed method is verified for application to real three-dimensional interconnects.**

Index Terms **— 3-D interconnect, cylindrical conduction mode basis function, electric field integral equation, multifunction method, proximity effect, skin effect, through hole vias, bonding wires.**

I. INTRODUCTION

The trend of integrating subsystems in a three-dimensional (3-D) packaging such as system-in-package (SiP) requires accurate and efficient electrical design of the 3-D interconnects in the packaging. A technical challenge of electrical design arises in the generation of the entire coupling model of over a thousand interconnects such as bond wires and through-hole vias (THV) since they are complicatedly coupled in vertical as well as in parallel, as shown in Figure 1. The other design challenge is in the construction of wideband interconnect model for the design of system packaging, which includes analog, radio-frequency (RF), and digital sub systems.

For these modeling issues, a trade off between modeling time and accuracy should be considered in existing methods.

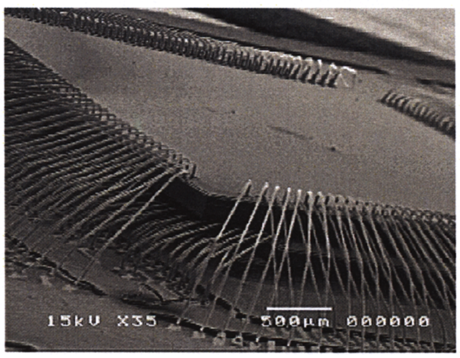

Figure 1. An example of 3-D bonding wire integration. (Photo courtesy of Amkor Technology, Inc.)

For example, equation-based methods provide parasitic values of interconnects with little computational effort [1-2], but they are not available for modeling of high-frequency effects such as skin and proximity effects. In contrast, various numerical methods can capture the high-frequency effects, but their application to the large and complicated 3-D interconnect problem requires much simulation time and memory.

To improve speed with maintaining accuracy in the 3-D interconnect modeling, we have proposed using the electric field integral equation (EFIE) combined with globally defined cylindrical conduction mode basis functions (CMBF) in [3-4]. The proposed method was found to be efficient for capturing high-frequency effects, compared with the conventional partial element equivalent circuit (PEEC) method [5] and the EFIE with CMBF's on the rectangular coordinate [6].

In addition to the previous research, this paper presents the acceleration of generating partial impedances by using the multifunction method (MFM), which uses various approximations in computing integrals for obtaining mutual inductances. MFM reduces computational burden for solving a system of large number of conductors because of the decrease in time for computing matrix elements and the sparse property of the higher-order mode sub matrix. For application examples, we simulated more practical structures such as THV array and vertically- and horizontally-coupled bonding wires.

II. OVERVIEW OF EFIE WITH CYLINDRICAL CMBF

The electrical modeling of interconnects is to find an equivalent circuit model that accurately describes frequency-dependent conduction losses and electromagnetic couplings. For this purpose, the classical PEEC method [5] and its variations are preferred since they extract the equivalent circuit model directly from the solution of the following volume EFIE.

$$\frac{\vec{J}(\vec{r},\omega)}{\sigma}+j\omega\frac{\mu}{4\pi}\int_{V'}G(\vec{r},\vec{r}\,')\vec{J}(\vec{r}\,',\omega)dV'=-\nabla\Phi(\vec{r},\omega), \quad (1)$$

where σ is conductivity, ω is angular frequency, μ is permittivity, and $G(\vec{r},\vec{r}\,')=1/|\vec{r}-\vec{r}\,'|$, which is Green's function with the retardation term neglected. For the approximation of the current density in (1), the CMBF-based methods use the following combination of global basis functions [4, 6].

$$\vec{J}_j(\vec{r},\omega) \approx \sum_{n,q} I_{jnq}\vec{w}_{jnq}(\vec{r},\omega), \qquad (2)$$

where

$$\vec{w}_{jnq}(\vec{r},\omega) = \begin{cases} \frac{\hat{z}_j}{A_{jn}} J_n(\alpha|\vec{r}-\vec{r}_j|\cdot\vec{\rho}_j)\cos n(\varphi_j-\varphi_{jq}) & \vec{r}\in V_j \\ 0 & \text{elsewhere} \end{cases}$$

is the (n, q) mode cylindrical CMBF in the j^{th} conductor, A_{jn} is the effective area for normalization, J_n is the n^{th} order Bessel function, and $\alpha^2 = -j\omega\mu\sigma$.

The cylindrical CMBF's are obtained from the solutions of diffusion equation of the current density on the circular cross section. Among the possible solutions of the equation, the fundamental-order (n=0) basis, which is independent of angular variable, is essential to describe skin effect of each conductor. If a conductor is significantly influenced by nearby conductors, the higher-order (n>0) bases are necessary to capture various proximity effects. In case of using the proximity effect basis of a certain order, two orthogonal (d- and q-) modes are sufficient to describe current crowding in arbitrary orientations [4].

By substituting (2) into (1) and applying the inner product based on the Galerkin's method, we can obtain the following equivalent circuit equation, which is composed of partial impedances and modal voltages.

$$\sum_{n,q} I_{jnq}R_{p,imd,jnq} + j\omega\sum_{n,q} I_{jnq}L_{p,imd,jnq} = V_{imd}, \qquad (3)$$

where

$$R_{p,imd,jnq} = \frac{1}{\sigma}\int_{V_i} \vec{w}^*_{imd}(\vec{r}_i,\omega)\cdot\vec{w}_{jnq}(\vec{r}_i,\omega)dV_i,$$

$$L_{p,imd,jnq} = \frac{\mu}{4\pi}\int_{V_i}\int_{V_j} \vec{w}^*_{imd}(\vec{r}_i,\omega)\cdot\vec{w}_{jnq}(\vec{r}_j,\omega)G(\vec{r}_i,\vec{r}_j)dV_jdV_i, \text{ and}$$

$$V_{imd} = -\int_{S_i} \Phi(\vec{r}_i)\vec{w}^*_{imd}(\vec{r}_i,\omega)d\vec{S}_i.$$

Calculating multiple integrals in the partial impedances can be a source of computational cost in the proposed method. However, the cost can be minimized by using analytic expressions and other techniques. At first, partial resistances matrix is diagonal because of the orthogonal property of the basis functions, and all the elements can be found from analytic expressions. In the case of calculating partial self inductances, a coordinate transform reduces a numerical integration, and efficiently controls singular points in Green's function. Computational cost in calculating partial mutual inductances is reduced by using the analytic expression for the integral over the axial variables. In addition, the multifunction method discussed in the next section finds smaller number of required higher order elements, so accelerates the formation of the global partial impedance matrix.

The modal voltage V_{imd} in (3) is simplified for different conduction modes. If the basis function is in the skin effect mode, the modal voltage is the same as the nodal voltage

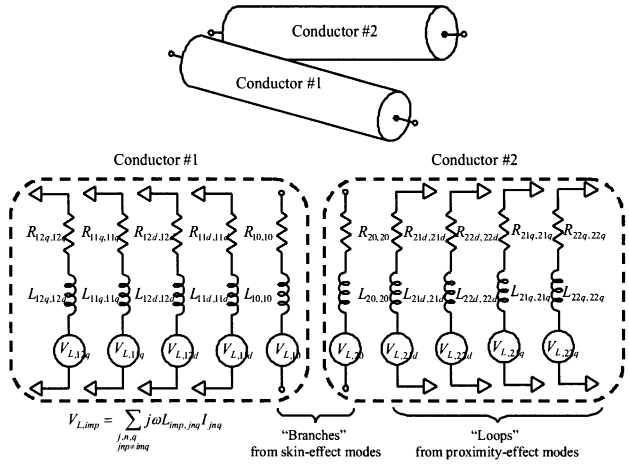

Figure 2. Equivalent circuit example for two conductor segments.

difference of the conductor segment since the surface integral with the basis function becomes unity. If the basis is in the higher-order skin effect modes, the integral becomes zero due to the harmonic function in the higher-order functions. Therefore, the equivalent circuit is composed as shown in Figure 2, where the components from the skin effect mode are connected to the physical nodes, and the other components from the proximity effect modes form closed loops.

III. Multifunction Method

For solving the problem of large number of interconnects, additional improvement of computation speed is useful. One idea of reducing the computational effort is applying the multifunction method (MFM) [7], which uses simple approximate formula or lower-order integrations for calculation of mutual inductances between sufficiently separated conductors. For example, thin-wire approximate formula, which is independent of the local coordinates of each conductor segment, can reduce computation time considerably.

A multifunction approach used for the CMBF-based method is to control the number of higher-order modes for two conductor segments. Although the required number of bases for a conductor segment is determined by the minimum distance from nearby conductors, all the bases need not be used for calculation of mutual inductances with far-separated conductors. That is, using smaller number of bases for conductors with large separation is sufficient for maintaining accuracy. An apparent benefit of the variable number of bases is the reduction of the computation time for calculating partial mutual inductances. In addition, the higher-order sub matrix of the partial impedance matrix becomes sparse, so we can save memory for storing non-zero elements.

Figure 3 shows the relative error of the coupling coefficients with adding higher-order proximity effect modes in two-parallel-conductor problem. As the spacing between conductor segments increases, the required number of higher-order bases becomes smaller. With a permissible relative error

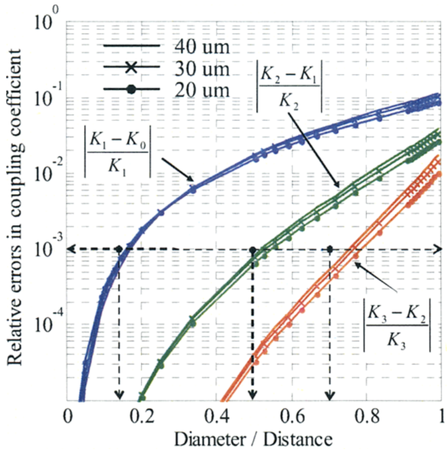

Figure 3. Relative errors in coupling coefficient at 10 GHz with adding higher-order modes in two-parallel-conductor configuration. The diameter of conductors varies from 20 to 40 microns.

(10^{-3} for example), we can determine thresholds of conductor distances (about 0.1, 0.5, and 0.7 for example) that find sufficient number of bases, as illustrated in Figure 3.

IV. Examples

A. Through Hole Via Array

Through hole via (THV) is an emerging interconnect structure in the 3-D packaging technology. A test example in this paper is shown in Figure 4, where ten-by-ten identical copper conductors are distributed with the same spacing. Nine diagonal conductors were selected to observe their frequency-dependent resistances and inductances, which are different due to the grounded conductor at an edge.

By applying MFM, required numbers of higher-order modes among conductor segments were determined based on the thresholds in Figure 3, resulting in the sparse higher-order-element part (Z_{pp}) in the global matrix, as shown in Figure 5.

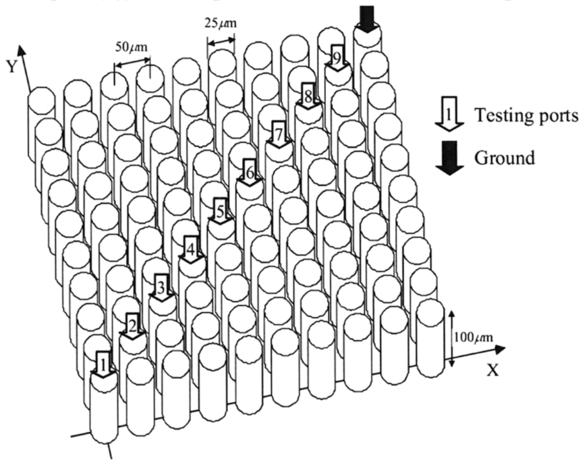

Figure 4. Geometry and port number indication of a ten-by-ten (a hundred) copper THV array.

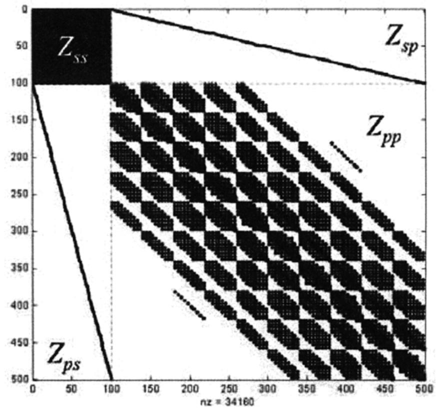

Figure 5. The sparsity pattern of global partial impedance of the ten-by-ten THV array model.

Figure 6 shows self resistances and inductances of the diagonal conductors. The parasitic values become diversing due to various proximity effects in high frequencies. That is, inductances of conductors near to the ground conductor (conductor # 9 for example) decrease more than other conductors far from the ground. Figure 7 also shows variations of mutual impedances in high frequencies. The unbalances in self and mutual parasitic components affect high frequency performance of 3-D packages.

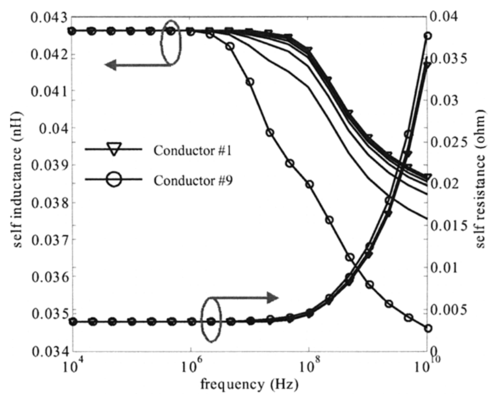

Figure 6. Self resistances and inductances of diagonal conductors in the THV array.

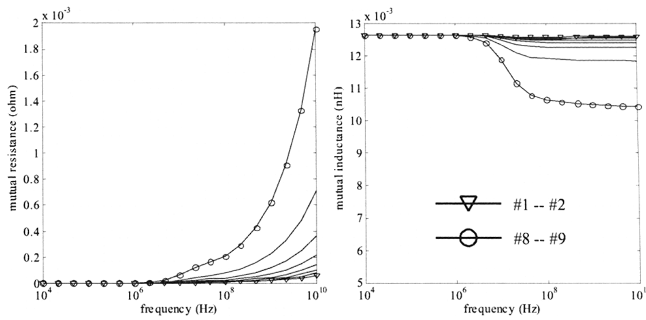

Figure 7. Mutual resistances and inductances between diagonal conductors that are separated to $50\sqrt{2}$ microns.

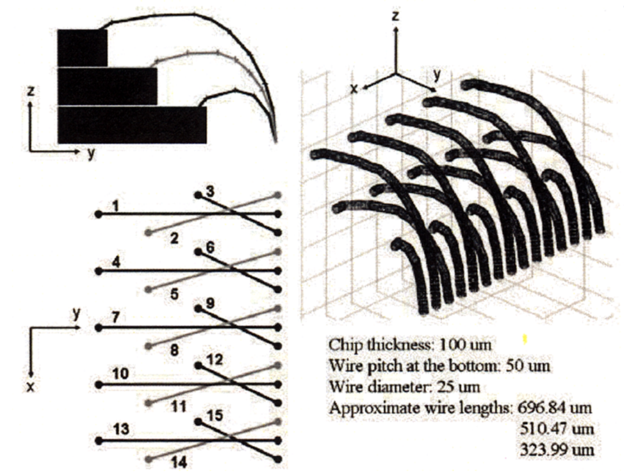

Figure 8. Geometry and port number indication of a 3-D fifteen-wire-bond example. (left upper: side view, left lower: top view, right: 3-D view.)

B. Horizontally and Vertically Coupled Bonding Wires

Bonding wires used in 3-D integrations connect ports on stacked IC's to printed traces on substrates. Wiring in high-density integration results in complicated configuration of large number of bonding wires, and finding coupling model of arbitrary oriented wires is a difficult task. To verify the capability of the proposed method in the 3-D bonding wire problem, we generated a fifteen bonding wire model on a three stacked IC's, as shown in Figure 8. Similar to the previous THV array example, frequency-dependent variations of resistances and inductances are obtained from the bonding wire example.

An interesting finding from the simulation is shown in Figure 9, where pairs of two wires form differential ports and three conductors are grounded. As known in design practice, the unbalanced ground as in Figure 9 (a) causes differences of inductances in high frequencies. Furthermore, we also observe a little difference of inductances in Figure 9 (b), where three center wires are grounded to maintain the balance. This imperfect balance comes from the geometric asymmetry of the three grounded conductors (#7 – #9), which provides different grounding conditions for two conductor pairs. This example shows that more deliberate design strategy is necessary for the electrical design of interconnects in the 3-D structure.

V. Conclusion

For modeling of large 3-D interconnects, this paper proposed an improvement in EFIE with cylindrical CMBF's. By using smaller number of higher-order basis for computation of mutual inductance, we could reduce the simulation time and memory for matrix storage. The application to the THV array and the 3-D bonding wires examples validates the efficiency of the proposed method for the wideband modeling of large interconnects in practice.

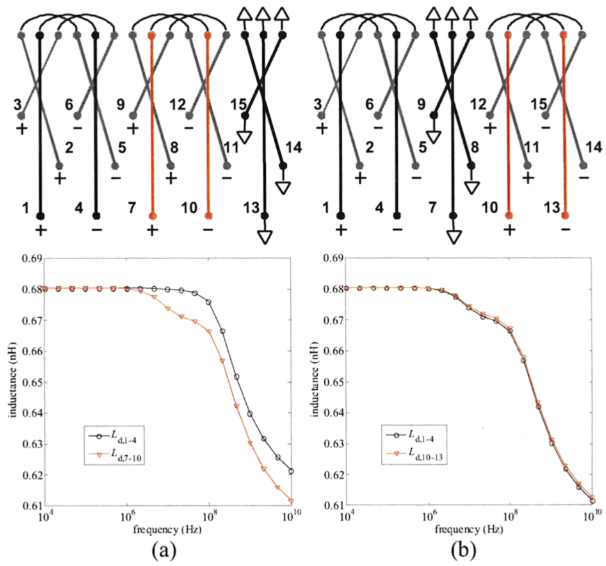

Figure 9. Differential inductances of wires for different ground wiring conditions. (a) Side wires are grounded. (b) Mid wires are grounded.

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