Parasitic Extraction of Interconnections in 3-D Packaging Using Mixed Potential Integral Equation with Global Basis Functions

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Introduction

As electronic packaging technology evolves with increasing integration density for wideband multi-media applications, the efficient generation of interconnection model becomes a critical part of electrical design. To achieve the efficient modeling of 3-D packaging interconnections, our previous research focused on the inductance calculation of large number of interconnections [1].

For extending the bandwidth of the interconnection model, this paper presents an equivalent circuit extraction method based on MPIE. Since the proposed method uses global basis functions to describe both current and charge densities, we can reduce the required memory of system matrix, which might be large in case that discretization is applied. Examples in this paper will validate the accuracy of the proposed method and show that the complexity of constructed equivalent circuit can be controlled according to the maximum modeling frequency.

MPIE Formulation with Global Basis Functions

Characterizing 3-D interconnection structure in free space is equivalent to solve Maxwell’s equation involving multiple conductors. The current and charge in each conductor contribute the electric potential of a testing point \( \vec{r} \), as shown in the following MPIE:

\[
\frac{j}{\sigma} \frac{\partial \vec{j}(\vec{r}, \omega)}{\partial t} + j \omega \frac{\mu}{4\pi} \int G(\vec{r}, \vec{r}') \frac{\partial \vec{j}(\vec{r}', \omega)}{\partial t} dV' = -\nabla \Phi(\vec{r}, \omega),
\]

\[
\frac{1}{4\pi \varepsilon_0} \int G(\vec{r}, \vec{r}') \rho(\vec{r}', \omega) dV' = \Phi(\vec{r}, \omega),
\]

where \( \vec{j}(\vec{r}, \omega) \) is current density, \( \rho(\vec{r}, \omega) \) is charge density, \( \Phi(\vec{r}, \omega) \) is electric potential, and \( G(\vec{r}, \vec{r}') \) is Green’s function. Retardation term in Green’s function is assumed to be negligible in this paper.

The MPIE can be directly converted to equivalent circuit model by using partial element equivalent circuit (PEEC) method [2] based on piecewise constant basis functions. Although the PEEC model is capable of describing DC and AC behaviors of interconnections, the discretization process is costly when we have to address large number of interconnections in 3-D integration.

To reduce the size of the system matrix and the corresponding equivalent network, global conduction mode basis functions (CMBF) can be used to...
capture high-frequency current density distribution [3]. For the application to 3-D interconnection problems that contain complicatedly coupled cylindrical conductors, cylindrical CMBFs are found to be efficient [1].

Similarly to the cylindrical CMBF for computing inductances and losses, we can define another global basis functions that capture charge density distribution on the surface of a cylindrical conductor as follows.

\[
v_{\text{ind}} = \begin{cases} \frac{1}{a_0} \cos(n \varphi - \varphi_{\text{ind}}) & \text{if } \vec{r} \in S_i, \\ 0 & \text{elsewhere} \end{cases}
\]

(2)

where \( n \) is the order of the basis function, \( S_i \) is the lateral surface of a conductor \( i \), and \( A_{\text{in}} \) is an effective area to normalize the surface integral of the basis functions. \( \varphi_{\text{ind}} \) represents the orientation of the basis function, which can be one of two orthogonal angles to describe charge density crowding caused by nearby conductors. The global basis functions for charges are obtained from the electric field solution of Laplace’s equation [4].

The initial step to extract equivalent model of a given interconnection is to define the maximum modeling frequency, which determines the required number of current and charge cells along the axial direction, as shown in Figure 1. Figure 1 also illustrates that the volume current cells and the surface charge cells are overlapped each other, so they construct the \( L-C \) network like the lumped-element approximation of transmission lines.

For each cell, current and charge are defined approximately with the linear combinations of global bases. Inserting the approximations to the MPIE (1) and applying inner product based on Galerkin’s method result in the following modal equivalent circuit equations.

\[
\sum_{m, n, q} I_{m, n, q} R_{m, n, q} + j \omega \sum_{m, n, q} I_{m, n, q} L_{m, n, q} = \Delta V_{\text{ind}} \\
\sum_{m, n, q} Q_{m, n, q} P_{m, n, q} = V_{\text{ind}}
\]

(3)

where \( i, j, k, \) and \( l \) are cell indices, and \( m, n, d, q, m', n', d', \) and \( q' \) are mode indices. Partial resistance \( R_{m, n, q} \) and inductances \( L_{m, n, q} \) are obtained from multiple volume integrals involving Green’s function and cylindrical CMBFs [1]. Similarly, partial coefficient of potential \( P_{m, n, q} \) is obtained from multiple surface integral involving Green’s functions and global basis functions (2). In addition to the above voltage equations, the following approximate form of the continuity equation (or KCL) for each node \( k \) should be considered to solve the entire equivalent circuit problem.

\[
\sum_{i} I_{i, 0} + j \omega Q_{k, 0} = 0.
\]

(4)

Although not shown for simplicity, the equivalent network in Figure 1 actually contains floating and grounded loops originated from the higher-order basis functions that capture proximity effect.
Results

As discussed in the previous research [1], the modal resistances and inductances in (3) effectively capture high-frequency partial resistances and inductances, which are affected by skin and proximity effects. Figure 2 shows a practical bonding wire examples on three stacked ICs, where partial resistances and inductances of the edge wires are different because of smaller proximity effects than inner wires.

Figure 3 shows scattering parameters of straight cylindrical conductor on the ideal ground. The effect of the ground was realized by an image conductor that is symmetric with respect to the ground plane. The increased number of cells is found to improve the accuracy of the equivalent model at the higher frequencies.

The dimensions of a bonding wire example in Figure 4 is obtained from the data in [5]. Inductance and capacitance cells are generated to make the cell length be less than 0.05 of the minimum wavelength (at 20 GHz). The magnitudes of scattering parameters in Figure 4 are well matched with the measurement data up to 10 GHz [5].

Conclusions

In conclusion, this paper presented an efficient method to extract the interconnection parasitic elements in 3-D packaging. The use of the MPIE with the global basis functions enables automatic generation of the equivalent circuit, complexity of which can be controlled by the defined model bandwidth. The proposed method is well applied several interconnection examples, and can be extended to modeling practical problems containing large number of interconnections.

References


Inductance (volume) cell
Capacitance (surface) cell

\[ V_{L,imp} = \sum_{j, j' \neq k} j V_{C,imp, jk} L_{j, k} \]

\[ V_{C,imp, jk} = \sum_{l, l' \neq k} P_{l, k} Q_{l, j} \]

Figure 1. Modal equivalent circuit model of a cylindrical interconnection.

Figure 2. 102 bonding wire geometry (upper) and their partial resistances and self inductances at 10 GHz (lower).

Figure 3. \( S \)-parameters of a copper cylinder on the ideal ground with different numbers of inductive cells. (length: 5 mm, diameter: 30 um, distance from cylinder center to ground: 50 um.)

Figure 4. Geometry (upper) and \( S \)-parameters (lower) of an aluminum bonding wire.