# Inductance and Resistance Calculations in Three-Dimensional Packaging Using Cylindrical Conduction-Mode Basis Functions

Ki Jin Han, Student Member, IEEE, and Madhavan Swaminathan, Fellow, IEEE

*Abstract*—For the successful electrical design of system-inpackage, this paper proposes an efficient method for extracting wideband resistance and inductance from a large number of 3-D interconnections. The proposed method uses the modal equivalent network from the electric field integral equation with cylindrical conduction-mode basis function, which reduces the matrix size for large 3-D interconnection problems. Additional enhancement schemes proposed further reduce the cost for computing the partial inductances. Therefore, the method discussed in this paper can be used to construct accurate models of a large number of 3-D interconnection structures such as more than 100 bonding wires used for stacking chips.

*Index Terms*—Bonding wires, cylindrical conduction-mode basis function (CMBF), electric field integral equation (EFIE), partial element equivalent circuit (PEEC) method, proximity effect (PE), skin effect (SE), system-in-package (SIP), through-hole via (THV) interconnections.

## I. INTRODUCTION

POPULAR choice for realizing miniaturized multimedia system in today's microelectronics is integrating various submodules in a single package. Compared to traditional multichip modules, modern package-based system achieves higher density of integration with the employment of stacked integrated-circuit (IC) technology, which is called 3-D integration or system-in-package (SIP). With the additional benefits of simple design and efficient IC processing [1], SIP is gaining popularity among package manufacturers since it offers better electrical performance through shorter interconnect lengths at lower cost. However, the commercialization of SIP is facing difficulty in achieving the desired electrical performance, which is due to the unexpected coupling and loss from complicated 3-D interconnections such as bonding wires (Fig. 1) and via interconnections [2]. Thus, efficient electrical modeling of 3-D interconnections is becoming a critical issue in SIP interconnection design.

The authors are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0250 USA (e-mail: kjhan@ece.gatech.edu; madhavan@ece.gatech.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCAD.2009.2016642



Fig. 1. Example of 3-D bonding wire integration (photograph courtesy of Amkor Technology, Inc.).

A major difficulty in modeling 3-D interconnections comes from the need to obtain the entire coupling model of a large number of 3-D interconnections. In a typical SIP that is composed of several stacked ICs, the number of bonding wires or through-hole via (THV) interconnections is close to a thousand [3], causing coupling between interconnections due to crisscrossing of the wires. Furthermore, the coupling model should include frequency-dependent losses caused by skin and proximity effects (SE and PE, respectively), which are particularly significant in high-frequency applications that require matched interconnection impedance. Therefore, for accurate electrical design of SIP covering wide frequency range for radio-frequency, analog, and digital applications, a frequencydependent coupling model of several interconnections is required.

For characterizing large 3-D interconnections, current modeling methods have limits in their applicability. For example, measurement-based methods are complicated due to the complexity of probing the 3-D structures [4]. Other available modeling approaches utilize analytical expressions of either partial inductances [5], [6] or segmented transmission lines [7] to extract the coupling model with low computational cost, but these simplified approaches do not capture high-frequency losses. Accurate high-frequency models are provided by numerical methods such as full-wave electromagnetic methods [8]–[10] or quasi-static parasitic extractor like FastHenry [11]. However, the use of these approaches is limited to solving

0278-0070/\$25.00 © 2009 IEEE

Manuscript received August 16, 2008; revised November 20, 2008. Current version published May 20, 2009. This work was supported by the Mixed Signal Design Tools (MSDT) Consortium of the Packaging Research Center, Georgia Tech, Atlanta, under Project 2126Q0R. This paper was recommended by Associate Editor H. E. Graeb.

mostly one or two interconnections because of their increased cost for solving large-size problems.

In order to address the current issues in modeling large 3-D interconnections, this paper proposes an efficient modeling method that extracts the equivalent network from the formulation of volume electric field integral equation (EFIE). The proposed method is based on the same framework as partial element equivalent circuit (PEEC) method [12], but it is different in the use of global conduction-mode basis function (CMBF). The original work [13] used the CMBF to improve efficiency for modeling planar interconnections. However, it was not suitable for cylindrical geometry modeling, which is necessary for characterizing cylindrical conductors arising in 3-D integration. Therefore, this paper utilizes another type of basis function called cylindrical CMBF.

The structure of this paper is organized as follows. Section II introduces the CMBF with its classification and discusses formulation of EFIE combined with the basis functions. The formulation procedure also includes the computation of partial resistances and inductances and the construction of equivalent circuits. Section III discusses implementation of the proposed method with two schemes to achieve the capability required for a large number of 3-D interconnections. Section IV shows several application examples that validate the accuracy and efficiency of the proposed method, followed by the conclusion in Section V.

#### II. FORMULATION OF EFIE WITH CYLINDRICAL CMBFS

This section introduces the cylindrical CMBF with its classification and applies the basis functions to the construction of equivalent circuit equation. Several techniques for computing partial resistances and inductances are discussed as well.

## A. Cylindrical CMBF

The main feature of the CMBF is that it globally describes the current density distribution in the cross section of a conductor. Using this global nature of the CMBF reduces the required number of basis functions, which can be large when using localized constant basis functions [12]. Clearly, the smaller number of bases has a merit for reducing the size of partial impedance matrix, as discussed in the use of the CMBF for rectangular geometries [13].

The cylindrical CMBFs are constructed from the following current density diffusion equation [14]:

$$\nabla \times \nabla \times \vec{J} + \alpha^2 \vec{J} = 0 \tag{1}$$

where  $\vec{J}$  is the current density (in amperes per square meter),  $\alpha^2 = -j\omega\mu\sigma = -((1+j)/\delta)^2$ ,  $\omega = 2\pi f$  is the angular frequency (in radians per second),  $\mu = 4\pi \times 10^{-7}$  is the freespace permeability (in henries per meter),  $\sigma$  is the conductivity (in siemens per meter), and  $\delta = 1/\sqrt{\pi f \mu \sigma}$  is the skin depth (in meters). One assumption of deriving (1) from Maxwell's equation is that the medium is a good conductor ( $\sigma \gg \omega \epsilon$ ). The other assumption about the current density is that it flows in the axial direction without any longitudinal variation. These assumptions are valid for thin conductors used in practice. By inserting  $\vec{J} = J_z(\rho, \varphi)\hat{z}$ , (1) is simplified to the following equation in cylindrical coordinates:

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial J_z}{\partial\rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 J_z}{\partial\varphi^2} + \alpha^2 J_z = 0.$$
(2)

By using the separation of variables, i.e.,  $J_z(\rho, \varphi) = R(\rho)\Phi(\varphi)$ , (2) is separated into the following two ordinary differential equations:

$$\rho^2 R''(\rho) + \rho R'(\rho) + (\alpha^2 \rho^2 - \nu^2) R(\rho) = 0$$
(3)

$$\Phi''(\varphi) + \nu^2 \Phi(\varphi) = 0. \tag{4}$$

Since the current density distribution should be continuous over the conductor cross section, the solutions of (4) are periodic (harmonic) functions, and  $\nu$  should be an integer n. Substituting  $\nu^2$  by  $n^2$  converts (3) into the Bessel differential equation of the order n. Therefore, the basis functions, which are the solutions of the diffusion equation, have the following form [15]:

$$\cos\left(n(\varphi - \varphi_0)\right) J_n(\alpha \rho), \qquad n = 0, 1, 2, \dots$$
 (5)

where  $J_n(\alpha \rho)$  is the *n*th-order Bessel function or Kelvin function [16], whose asymptotic behavior is the exponential function of  $\rho$ . For the use of (5) as basis functions, a proper classification of the order *n* and the orientation  $\varphi_0$  is necessary.

Physical behaviors of the bases with different orders classify the cylindrical CMBFs into SE and PE modes. The SE-mode basis is the fundamental order (n = 0) function, which shows the same behavior as that of the SE current distribution in a circular cross section. The PE modes are the remaining higher order (n > 0) basis functions, which have sinusoidal behaviors in their angular variations. The collection of the harmonic angular functions in PE modes enables the description of current crowding caused by PEs. Considering that Fourier series expansion can express any periodic function of  $\varphi$ , we classify two orthogonal basis functions for each order of the PE modes. In summary, the cylindrical CMBFs are classified to the following groups for the *i*th conductor in the global coordinates.

SE mode (n = 0):

$$\vec{w}_{i0} = \begin{cases} \frac{\hat{z}_i}{A_{i0}} J_0\left(\alpha(\vec{r} - \vec{r}_i) \cdot \hat{\rho}_i\right), & \vec{r} \in V_i \\ 0, & \text{elsewhere.} \end{cases}$$
(6)

PE direct (PE-d) mode (n > 0):

$$\vec{w}_{ind} = \begin{cases} \frac{\hat{z}_i}{A_{in}} J_n \left( \alpha(\vec{r} - \vec{r}_i) \cdot \hat{\rho}_i \right) \cos(n\varphi_i), & \vec{r} \in V_i \\ 0, & \text{elsewhere.} \end{cases}$$
(7)

PE quadrature (PE-q) mode (n > 0):

$$\vec{w}_{inq} = \begin{cases} \frac{\hat{z}_i}{A_{in}} J_n \left( \alpha(\vec{r} - \vec{r}_i) \cdot \hat{\rho}_i \right) \sin(n\varphi_i), & \vec{r} \in V_i \\ 0, & \text{elsewhere} \end{cases}$$
(8)

where  $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$  is a point in the *i*th conductor,  $\vec{r_i} = x_{i0}\hat{x} + y_{i0}\hat{y} + z_{i0}\hat{z}$  is the center point of the *i*th conductor, and  $A_{in}$  is the effective area.  $A_{in}$  is the constant that normalizes the basis function so that the integration of the function over the



Fig. 2. Examples of cylindrical CMBFs at  $10^8$  Hz and their combination to generate a current density distribution.

cross section equals unity. If n = 0, the effective area can be found as follows:

$$A_{i0} = \frac{2\pi\rho_i}{\alpha} J_1(\alpha\rho_i). \tag{9}$$

However, the integrations of the higher order basis functions  $(n \ge 1)$  should be zero due to the harmonic component in the  $\varphi$ -direction. Thus, the normalization is redefined as follows:

$$\int_{S_{in}} \vec{w}_{in} \cdot d\vec{S} = \frac{1}{2n} \tag{10}$$

where  $S_{in}$  is a part of the cross section occupied by a halfperiod of the harmonic function. By inserting (7) or (8)

$$A_{in} = \frac{2^{2-n} \alpha^n \rho_i^{2+n}}{(2+n)n!} {}_1F_2\left(1 + \frac{n}{2}; \left\{2 + \frac{n}{2}, 1+n\right\}; -\frac{1}{4} \alpha^2 \rho_i^2\right)$$
(11)

where  $_1F_2$  is one of the forms of the generalized hypergeometric functions.

A main advantage of using the cylindrical CMBF is that the orthogonal PE-mode bases automatically capture current crowding in any orientation. This feature makes the proposed method to be free from preconstructing the shapes of the basis functions based on conductor geometry or the generation of proximity templates [17]. Thus, we can apply the cylindrical CMBF for more general 3-D interconnection problems, where many conductor segments are located complicatedly. For example, Fig. 2 shows how the linear combination of SE- and PE-mode basis functions describes a specified current density distribution induced by the vicinity of nearby conductors.

### **B.** EFIE Formulation

The previously defined cylindrical CMBFs are inserted into volume EFIE to form equivalent voltage equations, which are



Fig. 3. General configuration of N-cylindrical conductor system.

composed of the modal partial impedances of each conductor. This section outlines the formulation procedure, including the calculation of the impedances and the construction of equivalent network.

1) Voltage Equation: As in the classical PEEC method [12] and the original CMBF-based method [13], the proposed method approximates the following volume EFIE:

$$\frac{\vec{J}(\vec{r},\omega)}{\sigma} + j\frac{\omega\mu}{4\pi}\int\limits_{V'} G(\vec{r},\vec{r'})\vec{J}(\vec{r'},\omega)dV' = -\nabla\Phi(\vec{r},\omega) \quad (12)$$

where  $\Phi(\vec{r}, \omega)$  is electric potential (in volts) and  $G(\vec{r}, \vec{r'}) = e^{-jk_0|\vec{r}-\vec{r_j}|}/|\vec{r}-\vec{r_j}|$  is Green's function. An assumption in this paper is that the maximum size of the problem space is much smaller than the wavelength of the maximum modeling frequency, so the retardation term  $(e^{-jk_0|\vec{r}-\vec{r_j}|})$  is negligible as in the  $(L_p, R)$  PEEC method [18].

Fig. 3 defines a general *N*-conductor system to be discussed in this section. As discussed in the previous section, a major difference of the CMBF-based method from the classical PEEC method is that the integral equation is not discretized to volume filaments but to globally defined conduction modes. However, each cylindrical CMBF is actually localized to each conductor, as shown in (6)–(8). Therefore, for the approximation of current density on a conductor segment j, basis functions that belong to the conductor are combined as follows:

$$\vec{J}_j(\vec{r},\omega) \cong \sum_{n,q} I_{jnq} \vec{w}_{jnq}(\vec{r},\omega).$$
(13)

By inserting the approximation (13) into the current density term in (12) and applying the following inner product based on Galerkin's method:

$$\langle \vec{w}_{imd}(\vec{r},\omega), \vec{x} \rangle = \int\limits_{V} \vec{w}_{imd}^*(\vec{r},\omega) \cdot \vec{x} dV \tag{14}$$

we obtain the voltage equation containing the following partial resistance, partial inductance, and modal voltage difference:

$$\sum_{n,q} I_{jnq} R_{imd,jnq} + j\omega \sum_{n,q} I_{jnq} L_{imd,jnq} = V_{imd}^j$$
(15)

where

$$\begin{split} R_{imd,jnq} &= \frac{1}{\sigma} \int\limits_{V_i} \vec{w}_{imd}^*(\vec{r},\omega) \cdot \vec{w}_{jnq}(\vec{r},\omega) dV_i \\ L_{imd,jnq} &= \frac{\mu}{4\pi} \int\limits_{V_i} \int\limits_{V_j} \vec{w}_{imd}^*(\vec{r},\omega) \cdot \vec{w}_{jnq}(\vec{r_j},\omega) \frac{1}{|\vec{r_i} - \vec{r_j}|} dV_j dV_i \\ V_{imd}^j &= - \int\limits_{S_i} \Phi_j(\vec{r_i}) \vec{w}_{imd}^*(\vec{r_i},\omega) \cdot d\vec{S_i}. \end{split}$$

After applying the same inner products with other basis functions in the *i*th conductor, we can combine all voltage equations into the following submatrix equation that represents the interactions between all modes in two conductors i and j:

$$(\mathbf{R}_{ij} + j\omega \mathbf{L}_{ij})\mathbf{I}_{j} = \mathbf{V}_{i}^{j}$$
(16)

where

$$\mathbf{R_{ij}} = \begin{pmatrix} R_{i0,j0} & R_{i0,j1d} & \cdots & R_{i0,jNq} \\ R_{i1d,j0} & R_{i1d,j1d} & \cdots & R_{i1d,jNq} \\ & & \ddots & \\ R_{iMq,j0} & R_{iMq,j1d} & \cdots & R_{iMq,jNq} \end{pmatrix}$$
$$\mathbf{L_{ij}} = \begin{pmatrix} L_{i0,j0} & L_{i0,j1d} & \cdots & L_{i0,jNq} \\ L_{i1d,j0} & L_{i1d,j1d} & \cdots & L_{i1d,jNq} \\ & & \ddots & \\ L_{iMq,j0} & L_{iMq,j1d} & \cdots & L_{iMq,jNq} \end{pmatrix}$$
$$\mathbf{I_j} = (I_{j0} & I_{j1d} & \cdots & I_{jNq})^{\mathrm{T}}$$
$$\mathbf{V_i^j} = (V_{i0}^j & V_{i1d}^j & \cdots & V_{iMq}^j)^{\mathrm{T}}.$$

Finally, all the submatrix equations between conductor segments congregate to form the global impedance matrix equation that contains loss and inductive coupling in the entire

 $D^2$ 

conductor system. The size of the global impedance matrix is approximately  $(N_c N_m) \times (N_c N_m)$ , where  $N_c$  and  $N_m$  are the number of conductor segments and the required number of modes for a conductor, respectively.  $N_m$  is one when only the SE mode is used and is more than one when additional PE modes are used. Since, practically, the sufficient number of PEmode basis pairs is two or three for the accurate description of current crowding, the required memory of the proposed method is considerably reduced compared to that of the classical PEEC method. Controlling the number of PE bases, to be discussed in Section III, further reduces the computational cost.

2) Partial Impedances: The calculations of partial resistances and inductances in (15), which are involved with sixfold integrals of frequency-dependent integrands, may be computationally expensive. In order to reduce calculation time, this section discusses analytical expressions and other numerical integration techniques.

For the partial resistances, indefinite integrals are easily found, and the mutual resistances vanish because of the local and orthogonal properties of the cylindrical CMBFs. Therefore, the global matrix of partial resistances becomes diagonal (see (17), shown at the bottom of the page). The partial resistance from the SE mode is actually identical to the analytic internal resistance formula of a cylinder [19].

In contrast to the partial resistances, fully closed form expressions of the partial inductances are not found, so a complicated numerical multiple integral is inevitable. However, analytic integrations over three variables reduce the original sixfold integral to the following triple integral:

$$L_{imd,inq} = \frac{\mu}{8\pi} \int_{0}^{\rho_i} \int_{0}^{\rho_i} \int_{0}^{2\pi} \rho \rho' \frac{J_m^*(\alpha \rho) J_n(\alpha \rho')}{A_{im}^* A_{in}} I_{\varphi_{\Sigma}} I_z d\varphi_{\Delta} d\rho' d\rho$$
(18)

where  $I_{\varphi_{\Sigma}}(\varphi_{\Delta})$ ,  $I_z(D, l_i)$ , and  $D^2(\rho, \rho', \varphi_{\Delta})$  are described in the equations found at the bottom of the page.  $D^2(\rho, \rho', \varphi_{\Delta})$ 

$$R_{imd,jnq} = \begin{cases} \frac{\pi \delta^2 \rho_i l_i}{\sigma |A_{i0}|^2} \Im \left( \alpha J_0^*(\alpha \rho_i) J_1(\alpha \rho_i) \right), & i = j, m = n = 0\\ \frac{\pi \delta^2 \rho_i l_i}{2\sigma |A_{im}|^2} \Im \left( \alpha^* J_{m-1}^*(\alpha \rho_i) J_m(\alpha \rho_i) \right), & i = j, m = n \neq 0, d = q\\ 0, & \text{otherwise} \end{cases}$$
(17)

$$I_{\varphi_{\Sigma}}(\varphi_{\Delta}) = \begin{cases} 8\pi - 4\varphi_{\Delta}, & m = n = 0\\ 2(2\pi - \varphi_{\Delta})\cos(n\varphi_{\Delta}) \pm \frac{2}{n}\sin(n\varphi_{\Delta}), & m = n \neq 0, d = q\\ \frac{4(-1)^{m+n+1}}{m^2 - n^2} \left[m\sin(m\varphi_{\Delta}) - n\sin(n\varphi_{\Delta})\right], & m \neq n, d = q(\text{PE} - d)\\ \frac{4(-1)^{m+n+1}}{m^2 - n^2} \left[n\sin(m\varphi_{\Delta}) - m\sin(n\varphi_{\Delta})\right], & m \neq n, d = q(\text{PE} - q)\\ 0, & \text{otherwise} \end{cases}$$
$$I_{z}(D, l_{i}) = 2\left(\sqrt{D^{2}} - \sqrt{l_{i}^{2} + D^{2}}\right) + l_{i}\log\left[\frac{l_{i}^{2} + \sqrt{l_{i}^{2} + D^{2}}}{-l_{i}^{2} + \sqrt{l_{i}^{2} + D^{2}}}\right]$$
$$(\rho, \rho', \varphi_{\Delta}) = \rho^{2} + \rho'^{2} - 2\rho\rho'\cos\varphi_{\Delta}$$



Fig. 4. Definition of the orientation parameters between the two separate cylinder segments.

is the distance between two points on the cross-sectional plane.  $\varphi_{\Delta}$  is a new angular variable that is obtained from the following coordinate transform of  $\varphi$  and  $\varphi'$ , which is useful for both reducing computational cost and avoiding singularity during numerical integrations [20]:

$$\begin{pmatrix} \varphi_{\Delta} \\ \varphi_{\Sigma} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \varphi \\ \varphi' \end{pmatrix}.$$
 (19)

Since Green's function is not a function of  $\varphi_{\Sigma}$ , an indefinite integral over  $\varphi_{\Sigma}$  reduces one of the six numerical integrals. In the remaining numerical integrals, singular points of the integrands are concentrated on a line where both  $\varphi_{\Delta} = 0$  and  $\rho = \rho'$  hold. In numerical integration based on adaptive Lobatto quadrature [21], the singular points are simply avoided by adjusting the starting points as a small value such as  $\varphi_{\Delta} = 0.01\pi$ .

For the calculation of the partial mutual inductances, we rewrite the inductance formula in (15) with the following frequency-dependent and frequency-independent parts:

$$L_{imd,jnq} = \frac{\mu}{4\pi} \int_{\rho_j,\rho_i} \rho_i \rho_j \frac{J_m^*(\alpha \rho_i) J_n(\alpha \rho_j)}{A_{imd}^* A_{jnq}} I_{z,\varphi} d\rho_i d\rho_j \quad (20)$$

where

$$\begin{split} I_{z,\varphi}(\rho_i,\rho_j) &= \int\limits_{\varphi_j,\varphi_i} \cos(\varphi_i - \varphi_{i,d}) \cos(\varphi_j - \varphi_{j,q}) \\ &\times \int\limits_{z_j,z_i} \frac{(\hat{z}_i \cdot \hat{z}_j)}{|\vec{r_i} - \vec{r_j}|} dz_i dz_j d\varphi_i d\varphi_j. \end{split}$$

The frequency-independent  $I_{z,\varphi}(\rho_i, \rho_j)$  is composed of an analytic integral over  $(z_i, z_j)$  and the other numerical integral over  $(\varphi_i, \varphi_j)$ .

For the calculation of the analytic integral over  $(z_i, z_j)$ , the distance between two points in two different conductor segments should be formulated with defined orientations. In Fig. 4, translations and rotations (based on Euler angles) can be determined from the defined global coordinates. Then, each point in a conductor is specified with the local cylindrical coordinates, which are actually related to the cylindrical CMBFs. With the calculated distance, we can obtain indefinite integrals for axial variables  $(z_i, z_j)$ , as discussed in Appendix A. This analytical expression is more generalized than that of the two thin conductor filaments [22] since it contains local coordinate variables that represent the inside of the two conductor segments.

To reduce the total frequency sweep time, the frequencyindependent integrals  $(I_{z,\varphi}$ 's) over  $(\varphi_i, \varphi_j)$  can be computed before a sweep simulation. The precomputed frequencyindependent  $I_{z,\varphi}$  is multiplied by the frequency-dependent integrands during the numerical integration over  $(\rho_i, \rho_j)$ . One issue of this approach is that storing the values of  $I_{z,\varphi}$ 's for every point of  $(\rho_i, \rho_j)$  requires a large amount of memory. Fortunately, we can reduce the memory requirement by using the property that  $I_{z,\varphi}(\rho_i, \rho_j)$  is a "smooth" bivariate function. That is, the variations in the frequency-independent part of the integrand are smaller than those in the frequency-dependent Kelvin functions. Therefore, after computing integration values for only a small number of data points, the following simple interpolation formula can be used:

$$I_{z,\varphi}(\rho_i,\rho_j) \simeq (1-s)(1-t)I_{p,q} + s(1-t)I_{p,q+1} + (1-s)tI_{p+1,q} + stI_{p+1,q+1}$$
(21)

where  $0 \le s$  and  $t \le 1$  are interpolation parameters and  $I_{p,q}$ 's are sampled points near  $(\rho_i, \rho_j)$ . Each  $I_{p,q}$  is obtained by a double numerical integral over  $(\varphi_i, \varphi_j)$  based on the adaptive Simpson quadrature rule. The total number of sampled points is determined adaptively according to the relative variation of Green's function.

For integrals over the remaining two variables ( $\rho_i$  and  $\rho_j$ ), the double integral using adaptive Lobatto quadrature [21] was used.

*3) Equivalent Circuit:* In addition to the calculated partial resistances and inductances, the following modal voltage difference should be considered to generate the global impedance matrix equation and the corresponding equivalent circuit:

$$V_{imd} = \sum_{j} V_{imd}^{j} \tag{22}$$

where  $V_{imd}^{\mathcal{I}}$ 's are modal voltages induced by the *j*th current density in (15). Since the integral over the lateral surface of a cylinder is zero, we can simplify  $V_{imd}$  to the integral over the inlet and the outlet planes  $(S_i^- \text{ and } S_i^+, \text{respectively})$  as follows:

$$V_{imd} = -\int_{S_{i}^{+}} \Phi(\vec{r_{+}}) \vec{w}_{imd}^{*}(\vec{r_{i}}, \omega) \cdot d\vec{S_{i}^{+}} -\int_{S_{i}^{-}} \Phi(\vec{r_{-}}) \vec{w}_{imd}^{*}(\vec{r_{i}}, \omega) \cdot d\vec{S_{i}^{-}}$$
(23)

where the potentials  $\Phi(\vec{r_+})$  and  $\Phi(\vec{r_-})$  are assumed to be constant over the cross sections.

When the SE-mode basis is involved, the integrals of  $\vec{w}_{imd}^*$  in (23) are unities since the basis functions are normalized, as

discussed in Section II. Thus, the modal potential difference becomes the actual voltage difference between the two nodes. In the case where the PE-mode bases are involved, the modal potential difference becomes zero since the integrals of the harmonic functions in the higher order bases vanish. In summary, the global impedance matrix equation is expressed as follows:

$$\begin{pmatrix} \mathbf{Z}_{ss} & \mathbf{Z}_{sp} \\ \mathbf{Z}_{ps} & \mathbf{Z}_{pp} \end{pmatrix} \begin{pmatrix} \mathbf{I}_{s} \\ \mathbf{I}_{p} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Delta} \mathbf{V}_{i} \\ \mathbf{0} \end{pmatrix}$$
(24)

where  $\mathbf{Z}_{ss}$ ,  $\mathbf{Z}_{sp}$ ,  $\mathbf{Z}_{ps}$ , and  $\mathbf{Z}_{pp}$  are partial impedances grouped by SE and PE modes;  $\mathbf{I}_s$  and  $\mathbf{I}_p$  are SE and PE currents, respectively; and  $\Delta \mathbf{V}_i$  is the voltage difference across a conductor segment.

In the viewpoint of circuit topology, the equivalent circuit generated from the PE-mode basis function forms a closed loop like a shielded conductor, which is inductively coupled with the other circuits. In an example of the equivalent circuit of two conductor segments [Fig. 5(a)], two branches are generated from the SE-mode partial components, and eight loops come from four orthogonal pairs of two PE modes. The number of PE-mode loops varies according to the strength of PE.

Extending the two-conductor model, Fig. 5(b) shows a general equivalent network of coupled 3-D bonding wires. The wires are approximated to the connections of several straight conductors, and the physically connected nodes are identical to the circuit nodes of the SE branches. During the approximation of the bonding wires with the conductor segment model, the number of segments is controlled so that the approximate model captures the original curvature of bonding wires accurately. Since the proposed method assumes that current flows in the axial direction only, current distribution may be inaccurate, particularly at any sharp edge connecting adjoining conductor segments.

#### **III. EFFICIENCY ENHANCEMENTS AND IMPLEMENTATION**

The proposed method discussed throughout the previous section has the benefit of the equivalent network's system matrix (24) being much smaller than the matrix of the classical PEEC method. However, the calculation of the partial impedances (15) for each frequency step is more complicated than the classical PEEC method. Therefore, for the modeling of real 3-D interconnections in SIP, the further reduction of computational cost is necessary.

#### A. Controlling the Number of PE-Mode Basis Functions

One of the ideas for reducing computational cost is to use the number of higher order (PE-mode) bases differently for each neighboring conductor [23]. We can assign reduced number of higher order bases to each conductor, because calculations related to the higher order basis functions are not necessary when the distance between two conductors is sufficiently large or when the coupling coefficient is small enough. This can be explained more clearly with an example [see Fig. 6(a)]. Here, we select an arbitrary conductor (e.g., conductor i) and group its neighboring conductors according to their different coupling



Fig. 5. Equivalent circuit examples, where two PE-mode basis functions are included for each conductor. (a) Equivalent circuit of two coupled cylinders. (b) Equivalent circuit of two bonding wires.

levels to the conductor *i*. For the group of conductors that are within the close proximity to the conductor *i*, such as conductor *j* in Fig. 6(a), the computation of PE-mode interactions is required up to the second order. However, for those that are more distant from conductor *i*, such as conductor *k*, only the computation of the first-order PE-mode interaction is required. Then, when generating the matrices involving PE-mode bases, only partial mutual inductances between the required PE modes are computed and filled, and other elements are set to zero, as shown in Fig. 6(b). Therefore, besides reducing time to compute modal mutual inductances, we can save memory for storing nonzero elements because such grouping enables the higher order submatrices ( $\mathbf{Z}_{sp}$ ,  $\mathbf{Z}_{ps}$ , and  $\mathbf{Z}_{pp}$ ) of the partial impedance matrix to become sparse.



Fig. 6. Efficiency enhancement schemes. (a) Conceptual diagram of two efficiency enhancement schemes. (b) Matrix  $(\mathbf{Z_{pp}})$  filling example involving the conductor *i* in (a). "X"'s and "0"'s represent the computed nonzero elements and zeros, respectively.

In the actual calculation of the required number of PE-mode basis functions in each group, we need to consider two determining key parameters. One is the initial coupling coefficient obtained with SE-mode bases only, and the other is the aspect ratio of the diameter to length of a cylinder. These parameters have the following important characteristics. The higher the initial coupling coefficient and the larger the aspect ratio are, the more higher order PE-mode basis functions are required. We can show these characteristics by drawing the boundaries of required number of basis functions under a defined error bound  $(10^{-3})$ , for example) in Fig. 7, which are obtained by computing relative errors in resultant coupling coefficients for various aspect ratios and initial coupling coefficients.

However, the numerical experiments of Fig. 7 are based on a simplified case where the two conductors are parallel and have an identical shape (thus, aspect ratio). Therefore, one might expect that more rigorous evaluation covering other possible cases where the two conductors have different orientation and shapes from each other is necessary. Nevertheless, using parallel conductors is a conservative way to find the required number of PE modes compared to other cases of arbitrarily oriented conductors, because PE is the most significant when the two conductors are in parallel. As for the issue of different shapes of cylinders, we can select the larger aspect ratio among the



Fig. 7. Relative error boundaries that define the required cylindrical CMBFs at 10 GHz (from copper two-parallel-conductor experiments.

two cylinders when determining the required number of PE modes.

## B. MFM

In addition to the PE-mode order reduction, we can use simplified approximate integrals for reducing the computational cost to generate  $\mathbf{Z}_{ss}$ . Since the approximations are frequency independent, the number of  $\mathbf{Z}_{ss}$  elements to be calculated is reduced during frequency sweep. Therefore, the computational effort of generating the dense matrix becomes that of generating a banded matrix.

When the two conductors are sufficiently separated (Fig. 6), the variations of current density in conductors are negligible. Thus, the following thin-filament approximation can be used instead:

$$L_{i,j} = \frac{\mu}{4\pi} \int_{z_i} \int_{z_j} G(\vec{r_i}, \vec{r_j}) dz_j dz_i.$$
<sup>(25)</sup>

The integrand in the aforementioned double integral does not contain frequency-dependent CMBFs and can be calculated in analytic way for any orientation of two straight conductor segments [22]. The accuracy of the thin-filament approximation is ensured when the distance between conductors is sufficiently large. The numerical experiments of two parallel cylinders with various dimensions show that the relative error of the thin-filament approximation from the exact integral depends on the aspect ratio of diameter to length of a cylinder, as in the case of the PE-mode order reduction. Fig. 8 shows the boundary where the thin-filament approximation is available for maintaining the relative error that is less than  $10^{-3}$ . The threshold pitch of using the frequency-independent approximations is usually higher than that of the controlling higher order bases.

For a conductor system occupying a very large dimension, the following center-to-center approximation [24] is also



Fig. 8. Error thresholds of using the thin-filament approximation as a function of diameter per length at 10 GHz.



Fig. 9. Flowchart of the 3-D inductance extraction tool.

available:

$$L_{i,j} = \frac{\mu}{4\pi} \frac{l_i l_j}{R_{ij}} \tag{26}$$

where  $l_i$ ,  $l_j$ , and  $R_{ij}$  are the lengths of the *i*th and the *j*th conductor, and the distance between the centers of the two conductors, respectively. From a similar numerical experiment, the relative pitch (w.r.t. length) where the center-to-center approximation is available is about 9.12, regardless of the cylinder lengths.

#### C. Implementation of Modeling Tool

Based on the discussed impedance calculation and efficiency enhancement schemes, we developed an inductance extraction tool called Interconnection Parasitic Extractor for 3-D integration (IPEX3D). The flowchart in Fig. 9 shows how the procedures of controlling PE-mode bases and multifunction method (MFM) are combined with the basic impedance computation routine.

In the beginning of the flowchart, the availability of using MFM is tested for every pair of conductors. If the distance



Fig. 10. Geometry of three parallel cylindrical copper conductors (three parallel aligned cylinders).

between the conductors is larger than a defined threshold distance, the thin-filament approximate integral can be computed for frequency-independent mutual inductance. If the distance is not large enough for the approximation, the sampled values of the frequency-independent integral  $I_{z,\varphi}$  are computed before the frequency sweep simulation.

During the frequency sweep, impedances from SE modes  $(\mathbf{Z}_{ss})$  are computed first. The initial coupling coefficients found from  $\mathbf{Z}_{ss}$  determine the required number of PE-mode basis functions by using the diagram in Fig. 7. The total number of PE-mode bases determines the sizes of  $\mathbf{Z}_{pp}$  and  $\mathbf{Z}_{sp}$ , whose values are computed at the latter part of the frequency sweep. Finally, conductor and wire impedances  $\mathbf{Z}_{c}$  and  $\mathbf{Z}_{w}$  are found from the matrix equation (24).

## IV. VALIDATION

In this section, the accuracy of the proposed method is validated from the comparison with existing simulation tools. Additionally, the efficiency of the proposed method is demonstrated with its applications to large interconnection structures. All simulations were performed using Intel Xeon 3-GHz CPU with 3.25-GB RAM.

## A. Accuracy Validation With Three Cylindrical Conductors

This section demonstrates simple three-conductor problems for evaluating the accuracy of the proposed approach. The test structure is shown in Fig. 10, where two conductors (1 and 2) are connected at the far end, with the other conductor being grounded. Since the accuracy of the proposed method should be examined for arbitrary orientations of conductor segments, we applied variations in rolling angle ( $\theta_R$ ), yawing angle ( $\theta_Y$ ), and parallel shift ( $L_s$ ) of conductor 1.

Loop resistances and inductances from these various situations are compared with the results from FastHenry [11], which is an inductance extracting tool based on the PEEC method combined with the fast multipole method. Since FastHenry uses brick-type filaments for modeling interconnection geometry, we constructed an approximate cylinder model with the brick elements, as shown in Fig. 11. For all the simulation cases, a logarithmic frequency sweep from  $10^4$  to  $10^{10}$  Hz was used, and the total number of frequency points is 31. Fig. 12 shows that the loop resistances and inductances obtained from the proposed method and FastHenry are well matched for all geometric variations. In Table I showing the relative accuracy



Fig. 11. Discretized approximate model of cylindrical conductors in FastHenry. The number of bricks per cross section is 378.



Fig. 12. Loop resistances and inductances of cylindrical conductors with geometric variations of conductor 1 (circles: FastHenry; lines: IPEX3D). (a) Loop inductances with increasing  $\theta_Y$ . (b) Loop inductances with increasing  $\theta_R$ . (c) Loop inductances with increasing  $L_s$ .

TABLE I COMPARISON OF IPEX3D AND FASTHENRY FOR THREE-CYLINDER PROBLEM

		Total		Maximum	
		simulation		relative	
		time (sec.) <sup><math>a</math></sup>		error $(\%)^b$	
		FastHenry	IPEX3D	loop R	loop L
	0	29065.0	860.9	3.01	0.92
	5	30213.5	1009.14	6.38	1.7
$\theta_R$ (deg.)	10	22023.1	944.79	5.64	1.46
	15	17633.1	710.57	4.81	1.17
	20	15571.6	601.47	4.14	0.94
	5	9240.05	725.88	5.68	0.28
$\theta_Y$ (deg.)	10	5225.37	608.65	4.83	0.15
	15	10444.6	604.6	5.0	0.15
	20	9488.7	609.0	5.15	0.22
	0.1	35840.1	878.22	5.23	2.19
$L_S \text{ (mm)}$	0.3	27670.5	510.22	7.59	2.87
	0.5	25928.3	616.84	11.37	2.88
	0.7	9134.4	509.40	1.76	0.19

of IPEX3D data (compared to FastHenry), the high-frequency error for loop resistances in the case of the shifted conductor is significant. It is because the conductor model used in IPEX3D does not accurately capture PE, which is concentrated in the overlapping regions of adjacent conductors. This error can be removed by increasing the number of segments along the axial direction (local Z-directions in Fig. 4).

As shown in Table I, IPEX3D requires much less simulation time than FastHenry mainly because the number of basis functions is considerably small. The large simulation times of FastHenry are due to the approximate discretization of the circular cross section, which may not be suitable for the optimal matrix computation, particularly at high frequencies. The number of required bases in IPEX3D varies with coupling levels for different conductor orientations. In this example, up to seven basis functions (one SE mode and six PE modes) were required. The simulation time also depends on the geometric configuration, but the effect is small because the integrations involving conductor orientations are not related to frequency.

## B. Scalability Analysis With THV Array

This section shows the required speed and memory for the modeling of the THV array in Fig. 13 with increasing number of interconnections for two different pitches (30 and 50  $\mu$ m). The THV array is a good configuration to perform the scalability analysis of the proposed method since it suffers from strong inductive couplings caused by the small pitch sizes. In addition, the electrical parameters of the THV array are useful for predicting the characteristics of emerging interconnection structures such as through-silicon-via and ball-grid-array (BGA) interconnections.

Before performing the scalability analysis, the resistances and inductances of a simple  $3 \times 3$  THV array with a pitch of 30  $\mu$ m were validated with FastHenry. Fig. 14 shows all the inductance and resistance values, and Table II compares the performance of the two simulators. As in the case of the previous section, the number of basis functions in IPEX3D is much smaller than that in FastHenry, so the simulation time of IPEX3D is reduced considerably. The possible sources of error (based on matrix norm) of IPEX3D relative to FastHenry,



Fig. 13. Copper THV array configuration.

which is less than 6.5%, as shown in Table III, come from the discretization of circular cross section in FastHenry. Table IV shows the required time for numerical integrations. The integration of self-inductances needs more effort than that of mutual inductances, but the total time for computing all the mutual elements takes more time. For both self-inductance and mutual inductance, integration time increases with frequency.

Since the dominant factor that determines the simulation speed of the proposed method is the time for generating the system matrix, the scalability analysis is focused on the measured values of the number of nonzero elements, as shown in Fig. 15(a) for two different pitches. Although the submatrix  $\mathbf{Z}_{ss}$  should be dense, controlling the number of PE-mode basis functions reduces the number of nonzeros in  $\mathbf{Z}_{pp}$  and  $\mathbf{Z}_{sp}$ . The number of nonzero elements directly influences the time for generating the system matrix in Fig. 15(b), which indicates that the computation time of the proposed method is between  $O(N^{1.6})$  and  $O(N^{1.8})$ , where N is the number of conductors.

The overall computational cost is actually a function of the strength of inductive coupling. Compared to the case of  $30\text{-}\mu\text{m}$  pitch, the increased pitch (50  $\mu\text{m}$ ) requires much less simulation time. Although today's design trend is to reduce the pitch size among interconnections, the pitch of 50  $\mu\text{m}$  between conductors with the interconnect diameter of 25  $\mu\text{m}$  is sufficiently small in current technology.

Fig. 16 shows the current density distribution of  $20 \times 20$  THV array with an "E"-shaped differential excitation. As frequency increases from  $10^7$  to  $10^9$  Hz, SE and PE become dominant, resulting in current crowding along the boundary of the differently excited conductors. Fig. 17 shows the resistances and inductances of 19 diagonal conductors, with an edge conductor being grounded. Different PEs and ground effects make a large variety of high-frequency resistances and inductances.

#### C. Large 3-D Bonding Wire Interconnection

In this section, the parasitic elements of close-to-real bonding wires on three stacked ICs are extracted. The bonding wire structure in Fig. 18 is obtained approximately from the model of



Fig. 14. Resistances and inductances of conductors in  $3 \times 3$  THV array (circles: FastHenry; lines: IPEX3D). (a) Conductor index and ground definition. (b) Resistances. (c) Inductances.

TABLE II Comparison of IPEX3D and Fasthenry for  $3 \times 3$  THV Array Problem

	FastHenry	IPEX3D
Number of frequency points	19	19
Number of basis functions <sup>a</sup>	378	5
Total simulation time (sec.)	7624.15	1199.27

TABLE III Relative Matrix Errors of IPEX3D to Fasthenry (in Percent) for  $3 \times 3$  THV Array Problem

	R	L
Average	1.52829	0.38416
Maximum	6.5434 (10 GHz)	1.3448 (10 GHz)



TABLE IV NUMERICAL INTEGRATION TIME OF IPEX3D (IN SECONDS PER ELEMENT) FOR  $3 \times 3$  THV Array Problem

Fig. 15. Scalability analysis of the proposed method with THV array model. The number of frequency points is 30. (a) Number of nonzero elements (left: pitch of 30  $\mu$ m; right: pitch of 50  $\mu$ m). (b) Total relative simulation time.

a triple-chip stacked chip-scale package [25]. The wire diameter is 25  $\mu$ m, and the pitch between adjacent wires is 60 or 80  $\mu$ m, which comes from an estimation of the state of the art [26]. All wires are in parallel. The approximate lengths of wires are 1.26 mm (class 1), 0.82 mm (class 2), and 0.48 mm (class 3), respectively.

For validation, the inductance and resistance values of six bonding wire structures in Fig. 19(a) were extracted, and the S-parameters from the R-L equivalent circuit model were compared with the values from CST Microwave Studio (MWS), a commercial full-wave EM simulator [27]. In the simulation setup, six ports were defined, as shown in Fig. 19(a), which utilizes each wire as a signal or a ground wire. Simulation results in Fig. 19(b) and (c) indicate good correlation, except at high frequencies. This high-frequency error can be attributed to the quasi-static assumption used in IPEX3D, which does not include capacitive coupling. Since bonding wires are dominated by inductive coupling, the absence of capacitive coupling is expected to have a small effect on typical structure. The other source of error is the approximation of conductor segments.



Fig. 16. Current density distribution at different frequencies in  $20 \times 20$  THV array. Excitation voltages to the dark and light parts of (a) are -1 and +1 V, respectively. (a) Excitation distribution. (b)  $10^7$  Hz. (c)  $10^8$  Hz. (d)  $10^9$  Hz.



Fig. 17. Resistances and inductances of diagonal conductors in  $20 \times 20$  THV array. (a) Conductor index and ground definition. (b) Resistances. (c) Inductances.

As discussed in Section II-B-3, the 1-D current assumption in IPEX3D may be inaccurate, particularly at the sharp edges of adjoining conductor segments. For example, the sharp edge



Fig. 18. Side view and segmentation geometry (in micrometers) of gold bonding wires on three stacked ICs.



Fig. 19. *S*-parameters of six bonding wires (circles: IPEX3D; lines: CST MWS). (a) Six-bonding-wire structure and port configuration. (b) Insertion losses (left:  $60 \mu m$  pitch; right:  $80 \mu m$  pitch). (c) Return losses (left:  $60 \mu m$  pitch; right:  $80 \mu m$  pitch).

shown in class 3 wire of Fig. 18 will generate some high-frequency error. The two combined error effects translate to a maximum error of 15.1% for  $|S_{3,3}|$  in Fig. 19(c), but this maximum error is acceptable because it occurs at small return loss values.



Fig. 20. Extracted bonding wire parasitics at 10 GHz, with a wire of class 2 being grounded. (a) 102-bonding-wire structure with index. (b) Wire resistances. (c) Wire self-inductances.

The final example consists of 102 bonding wires, as shown in Fig. 20(a), where 34 wires are mounted for each stack. Fig. 20(b) and (c) shows the resistances and self-inductances of all bonding wires at 10 GHz, with one conductor of class 2 being grounded. The grounded conductor influences on adjacent wires, as well as the upper and lower wires, so the resultant high-frequency resistances and inductances become different. Fig. 20 also shows small variations of wire impedances in the edge of the structure, whose PEs are smaller than those in the middle of the structure. Capturing these parasitic variations at high frequencies is useful for an efficient design of high-density 3-D interconnections for wideband applications.

# V. CONCLUSION

This paper presented an efficient method to extract frequency-dependent resistance and inductance from a large number of interconnections that are used in today's 3-D packaging. Unlike currently available methods, the proposed method improves the efficiency by using cylindrical CMBFs, whose global property reduces the size of the system matrix. In addition, the orthogonal property of the cylindrical CMBFs enables automatic capture of current crowding caused by SE and PE. This paper discussed the application of such cylindrical CMBFs to EFIE and provided details for computing partial resistances and inductances. Furthermore, we introduced two enhancement schemes accelerating the computation of mutual inductances, so that the speed to fill the system matrices is improved to  $O(N^{1.8})$ , where N is the number of conductors. Finally, our developed modeling tool based on the proposed method demonstrated good accuracy and capability for solving large 3-D interconnection structures. Therefore, the proposed method can be a possible solution to the industrial need for wideband electrical modeling of practical high-density interconnections arising in SIP or 3-D integration.

# APPENDIX A Indefinite Integral for Axial Variables in Mutual Inductance Formula

With the variables that are defined in Fig. 4, the distance between two points is formulated as follows:

$$R_{12} = |R_2 - R_1|$$
  
=  $\sqrt{z_1^2 + 2bz_1z_2 + z_2^2 + 2dz_1 + 2fz_2 + g}$  (27)

where

$$\begin{split} b &= -\sin\beta_1 \sin\beta_2 \cos(\alpha_2 - \alpha_1) - \cos\beta_1 \cos\beta_2 \\ d &= \rho_{2x} \left[ \sin\beta_1 \sin(\alpha_2 - \alpha_1) \right] \\ &+ \rho_{2y} \left[ \sin\beta_1 \cos\beta_2 \cos(\alpha_2 - \alpha_1) - \sin\beta_2 \cos\beta_1 \right] \\ &- D_x \sin\beta_1 \sin\alpha_1 + D_y \cos\alpha_1 \sin\beta_1 - D_z \cos\beta_1 \\ f &= \rho_{1x} \left[ -\sin\beta_2 \sin(\alpha_2 - \alpha_1) \right] \\ &+ \rho_{1y} \left[ \cos\beta_1 \sin\beta_2 \cos(\alpha_2 - \alpha_1) - \sin\beta_1 \cos\beta_2 \right] \\ &+ D_x \sin\beta_2 \sin\alpha_2 - D_y \cos\alpha_2 \sin\beta_2 + D_z \cos\beta_2 \\ g &= D_{21}^2 + \rho_1^2 + \rho_2^2 + 2 \\ &\times \left[ -\rho_{1x}\rho_{2x} \cos(\alpha_2 - \alpha_1) - \rho_{1y}\rho_{2y} \\ &\times (\cos\beta_1 \cos\beta_2 \cos(\alpha_2 - \alpha_1) + \sin\beta_1 \sin\beta_2) \\ &- \rho_{1y}\rho_{2x} \cos\beta_1 \sin(\alpha_2 - \alpha_1) \\ &+ \rho_{1x}\rho_{2y} \cos\beta_2 \sin(\alpha_2 - \alpha_1) \right] \\ &+ 2\rho_{1x} \left[ -D_x \cos\alpha_1 - D_y \sin\alpha_1 \right] + 2\rho_{1y} \\ &\times \left[ D_x \cos\beta_1 \sin\alpha_1 - D_y \cos\alpha_1 \cos\beta_1 - D_z \sin\beta_1 \right] \\ &+ 2\rho_{2x} \left[ D_x \cos\beta_2 \sin\alpha_2 + D_y \cos\alpha_2 \cos\beta_2 + D_z \sin\beta_2 \right]. \end{split}$$

 $\rho_{nx} = \rho_n \cos \varphi_n, \ \rho_{nx} = \rho_n \sin \varphi_n, \ \text{and} \ \alpha_n, \ \beta_n \ \text{are the rotation}$ angles of conductors based on Euler angles (n = 1, 2).

Plugging (27) into the integral over  $(z_i, z_j)$  in (20) and finding the indefinite integral result in the following formula:

$$I_{z}(\rho_{i},\rho_{j},\varphi_{i},\varphi_{j}) = \int_{z_{i},z_{j}} \frac{1}{R_{i,j}} dz_{i} dz_{j} = I_{z1} - I_{z2}$$
(28)

where

$$\begin{split} I_{z1} &= I(B_1, C_1, D_1, -bL_2/2 + d + L_1/2 \\ &+ bL_2/2 + d + L_1/2) \\ I_{z2} &= I(B_2, C_2, D_2, -bL_2/2 + d - L_1/2 \\ &+ bL_2/2 + d - L_1/2) \\ I(B, C, D, x^-, x^+) \\ &= \frac{1}{b} \int_{x^-}^{x^+} \log \left[ x + C\sqrt{(x-B)^2 + D^2} \right] dx \\ B_1 &= +(1-b^2) \frac{L_1}{2} - bf + d \\ B_2 &= -(1-b^2) \frac{L_1}{2} - bf + d \\ B_2 &= -(1-b^2) \frac{L_1}{2} - bf + d \\ C_1 &= C_2 = \left| \frac{1}{b} \right| = |\sec \theta_0| \\ D_1^2 &= b^2 \left\{ \frac{1-b^2}{4} L_1^2 + (d-bf) L_1 + g - f^2 \right\} \\ D_2^2 &= b^2 \left\{ \frac{1-b^2}{4} L_1^2 - (d-bf) L_1 + g - f^2 \right\} \end{split}$$

Detailed analytic expressions of  $I(B, C, D, x^-, x^+)$ , which depend on the specific values of B, C, and D, are not shown in this paper because of limited space.

### ACKNOWLEDGMENT

The authors would to thank A. E. Engin of San Diego State University, San Diego, CA, and D. Chung and S. H. Min of Georgia Institute of Technology, Atlanta, for their valuable theoretical pieces of advice.

#### REFERENCES

- R. R. Tummala, "SOP: What is it and why? A new microsystemintegration technology paradigm-Moore's law for system integration of miniaturized convergent systems of the next decade," *IEEE Trans. Adv. Packag.*, vol. 27, no. 2, pp. 241–249, May 2004.
- [2] E. Awad, H. Ding, R. S. Graf, and J. J. Maloney, "Stacked-chip packaging: Electrical, mechanical, and thermal challenges," in *Proc. IEEE 54th Electron. Compon. Technol. Conf.*, Jun. 2004, vol. 2, pp. 1608–1613.
- [3] M. Dreiza, A. Yoshida, J. Micksch, and L. Smith. (2005, Jul.) Stacked package-on-package design guidelines. *Chip Scale Rev.* [Online]. Available: http://www.chipscalereview.com/issues/0705/article.php?type= feature&article=f3
- [4] C. T. Tsai, "Package inductance characterization at high frequencies," *IEEE Trans. Compon., Packag., Manuf. Technol. B*, vol. 17, no. 2, pp. 175– 181, May 1994.
- [5] H. Patterson, "Analysis of ground bond wire arrays for RFICs," in *Proc. IEEE RFIC Symp. Dig.*, Jun. 1997, pp. 237–240.
- [6] F. W. Grover, Inductance Calculations—Working Formulas and Tables. Mineola, NY: Dover, 1946.
- [7] F. Alimenti, P. Mezzanotte, L. Roselli, and R. Sorrentino, "Modeling and characterization of the bonding-wire interconnection," *IEEE Trans. Microw. Theory Tech.*, vol. 49, no. 1, pp. 142–150, Jan. 2001.

859

- [8] C. Schuster, G. Leonhardt, and W. Fichtner, "Electromagnetic simulation of bonding wires and comparison with wide band measurements," *IEEE Trans. Adv. Packag.*, vol. 23, no. 1, pp. 69–79, Feb. 2000.
- [9] I. Doerr, L.-T. Hwang, G. Sommer, H. Oppermann, L. Li, M. Petras, S. Korf, F. Sahli, T. Myers, M. Miller, and W. John, "Parameterized models for a RF chip-to-substrate interconnect," in *Proc. IEEE 51st Electron. Compon. Technol. Conf.*, May/Jun. 2001, pp. 831–838.
- [10] J. Y. Chuang, S. P. Tseng, and J. A. Yeh, "Radio frequency characterization of bonding wire interconnections in a modeled chip," in *Proc. IEEE* 54th Electron. Compon. Technol. Conf., Jun. 2004, vol. 1, pp. 392–399.
- [11] M. Kamon, M. J. Tsuk, and J. K. White, "FastHenry: A multipoleaccelerated 3D inductance extraction program," *IEEE Trans. Microw. Theory Tech.*, vol. 42, no. 9, pp. 1750–1758, Sep. 1994.
- [12] A. E. Ruehli, "Equivalent circuit models for three-dimensional multiconductor systems," *IEEE Trans. Microw. Theory Tech.*, vol. MTT-22, no. 3, pp. 216–221, Mar. 1974.
- [13] L. Daniel, A. Sangiovanni-Vincentelli, and J. White, "Using conduction mode basis functions for efficient electromagnetic analysis of on-chip and off-chip interconnect," in *Proc. ACM/IEEE 38th Des. Autom. Conf.*, Jun. 2001, pp. 563–566.
- [14] P. Silvester, "Modal network theory of skin effect in flat conductors," *Proc. IEEE*, vol. 54, no. 9, pp. 1147–1151, Sep. 1966.
- [15] K. J. Han, M. Swaminathan, and E. Engin, "Electric field integral equation with cylindrical conduction mode basis functions for electrical modeling of three-dimensional interconnects," in *Proc. ACM/IEEE 45th Des. Autom. Conf.*, 2008, pp. 421–424.
- [16] M. Abramowitz and A. Stegun, Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables. New York: Dover, 1965.
- [17] L. Daniel, A. Sangiovanni-Vincentelli, and J. White, "Proximity templates for modeling of skin and proximity effects on packages and high frequency interconnect," in *Proc. IEEE/ACM Int. Conf. Comput.-Aided Des.*, Nov. 2003, pp. 326–333.
- [18] A. E. Ruehli and A. C. Cangellaris, "Progress in the methodologies for the electrical modeling of interconnects and electronic packages," *Proc. IEEE*, vol. 89, no. 5, pp. 740–771, May 2001.
- [19] C. R. Paul, *Analysis of Multiconductor Transmission Lines*. New York: Wiley, 1994.
- [20] S. Ortiz and R. Suaya, "Efficient implementation of conduction modes for modelling skin effect," in *Proc. IEEE Comput. Soc. Annu. Symp. VLSI ISVLSI*, Mar. 2007, pp. 500–505.
- [21] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing*, 2nd ed. Cambridge, U.K.: Cambridge Univ. Press, 1995.
- [22] S. Ray, A. K. Gogoi, and R. K. Mallik, "Closed form expression for mutual partial inductance between two skewed linear current segments," in *Proc. Antennas Propag. Soc. Int. Symp.*, Jul. 1999, pp. 1278–1281.
- [23] K. J. Han, M. Swaminathan, and E. Engin, "Wideband electrical modeling of large three-dimensional interconnects using accelerated generation of partial impedances with cylindrical conduction mode basis functions," in *Proc. IEEE MTT-S Int. Microw. Symp.*, 2008, pp. 1297–1300.
- [24] G. Antonini and A. E. Ruehli, "Fast multipole and multifunction PEEC methods," *IEEE Trans. Mobile Comput.*, vol. 2, no. 4, pp. 288–298, Oct.–Dec. 2003.

- [25] Y. Fukui, Y. Yano, H. Juso, Y. Matsune, K. Miyata, A. Narai, Y. Sota, Y. Takeda, K. Fujita, and M. Kada, "Triple-chip stacked CSP," in *Proc. IEEE 50th Electron. Compon. Technol. Conf.*, May 2000, pp. 385–389.
- [26] E. Milke, T. Mueller, and A. Bischoff, "New bonding wire for fine pitch applications," in *Proc. IEEE Electron. Packag. Technol. Conf.*, 2007, pp. 750–754.
- [27] Microwave Studio, Comput. Simul. Technol., Darmstadt, Germany, 2008. Version 2008.06.



**Ki Jin Han** (S'06) received the B.S. (*summa cum laude*) and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1998 and 2000, respectively. He is currently working toward the Ph.D. degree in the School of Electrical and Computer Engineering (ECE), Georgia Institute of Technology, Atlanta.

From 2000 to 2005, he was with the System Research and Development Laboratory, LG Precision (currently named LIG Nex1), Korea, where he was involved in the development of RF/microwave

transceiver and antenna system for airborne and naval radars. He is currently a Graduate Research Assistant with the School of ECE, Georgia Institute of Technology. His current research interests include computational electromagnetics, signal/power integrity for high-speed digital design, and the electromagnetic modeling of electronic packaging and interconnections.



**Madhavan Swaminathan** (A'91–M'95–SM'98– F'06) received the M.S. and Ph.D. degrees in electrical engineering from Syracuse University, Syracuse, NY, in 1989 and 1991, respectively.

He is currently the Joseph M. Pettit Professor in Electronics with the School of Electrical and Computer Engineering, Georgia Institute of Technology (Georgia Tech), Atlanta. He was the Deputy Director of the Microsystems Packaging Research Center, Georgia Tech, from 2004 to 2008. He is the Cofounder of Jacket Micro Devices. Inc., Atlanta, a

leader in integrated RF modules and substrates for wireless applications. Prior to joining Georgia Tech, he was with IBM, working on the packaging for supercomputers. He is the author of more than 300 journal and conference publications and the holder of 15 patents. He is the primary author of the book entitled "Power Integrity Modeling and Design for Semiconductors and Systems" (Prentice Hall, 2007) and a Coeditor of "Introduction to System on Package" (McGraw-Hill, 2008).

He has been recognized for his research through several awards, including the 2007 Technical Excellence Award from Semiconductor and Global Research Corporation (SRC/GRC).