Accelerated Frequency Domain Analysis by Susceptance-Element Based Model Order Reduction of 3D Full-wave Equations

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***Abstract* — A circuit-equivalent frequency-domain three-dimensional electromagnetic simulation for package structures is proposed. A robust and passive susceptance-element based model order reduction is applied to the governing equation for accelerated simulation and proof-of-concept is shown with the examples of power-ground structure simulations.**

I. Introduction

The analysis of power-ground structures presents a significant problem for signal/power integrity, as well as for electromagnetic interference (EMI) considerations [1] [2]. Such problems have been analyzed in the past by making use of time- and frequency-domain techniques. Most of these solvers can be classified, based on the degrees of freedom, as two-dimensional (2D), two-point-five dimensional (2.5D) and three-dimensional (3D). Though 3D solvers are the most accurate, they also impose a heavy penalty in terms of time and memory required for analysis. This problem can be alleviated by means of model order reduction approaches. The asymptotic waveform evaluation (AWE) [3] and Pade' via Lanczos (PVL) [4] algorithms have been previously applied for the analysis of interconnect structures. However, ensuring passivity was a bottleneck for these processes, which was addressed by the Passive Reduced-order Interconnect Macromodeling Algorithm (PRIMA) [5]. An analysis of the application of these techniques as applied to the 2D compact finite-difference frequency domain (FDFD) formulation is shown in [6]. However, PRIMA does not guarantee reciprocity among the ports. The Efficient Nodal Order Reduction (ENOR) [7] algorithm overcomes this problem by performing reduction of the governing equation in the nodal analysis form. It applies an orthogonal projection on the system based on moment-matching techniques in combination with Arnoldi-like orthogonalization [8].

This paper builds on the use of a full-wave simulation method by converting the Maxwell's equations into an electrical equivalent network [9], which has been validated. This offers the advantages of 1) making use of Spice-based circuit solvers to run full-wave simulations and 2) using circuit based numerical techniques to speed-up the simulation. An improved ENOR (imp-ENOR) algorithm [10] is then applied to the nodal formulation of this equivalent circuit resulting in a model-order reduced state equation. The system response can then be obtained in a fast memory-efficient manner as compared to the simulation of the full 3D problem. The paper is organized as follows: Section II describes the formulation of the equivalent circuit based simulator and the application of the imp-ENOR. Section III details the results and discussion.

II. Formulation

Consider the differential form of Maxwell's equation in the frequency domain

$$\nabla \times \mathbf{E} = -j\omega\mathbf{B} \tag{1}$$

$$\nabla \times \mathbf{H} = j\omega\mathbf{D} \tag{2}$$

where, **E** and **H** are the vector electric and magnetic fields
D and **B** are the vector electric and magnetic field densities
ω is the frequency in radians

Assuming an isotropic, lossless and homogeneous medium, the above equations can be written for a 2D transverse magnetic (TM) wave as:

$$j\omega\varepsilon E_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z \tag{3}$$

$$j\omega\mu H_x = -\frac{\partial E_z}{\partial y} \tag{4}$$

$$j\omega\mu H_y = \frac{\partial E_z}{\partial x} \tag{5}$$

where, ε and μ are the material permittivity and permeability, respectively

E_p and H_p represent the electrical and magnetic field in the p-direction (p = x or y).

J_z is the external current source in the z-direction

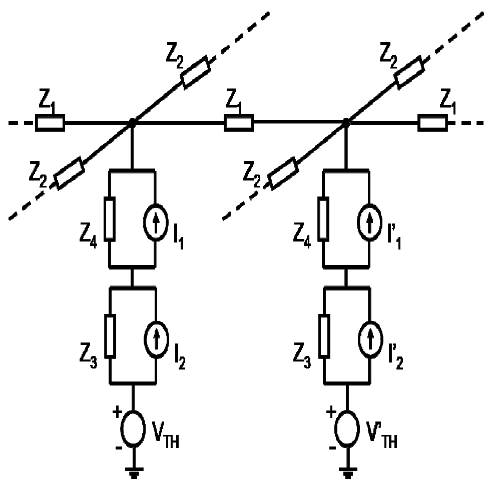


Fig.1 Equivalent-circuit unit-cell for 2D simulation

Discretizing the above equations using the Yee-grid, so as to implicitly satisfy the divergence laws, we can form an electrical equivalent circuit for the resulting equations as shown in Fig. 1. The nodal voltages represent the electrical fields and the magnetic fields map to the branch currents. The circuit branch connected to ground can be further simplified to an equivalent Norton circuit, thus reducing the problem to one of solving only for the nodal voltages. The solution of the electrical network results in a linear equation of form **Ax = b,** where **A** is the sparse and banded nodal analysis (NA) amplification matrix, **x** is the vector of unknown nodal voltages and **b** is the vector containing external current sources.

The current-controlled current sources (CCCS) are converted into voltage-controlled current sources (VCCS). The controlled sources can also be viewed in terms of mutual susceptances between the voltage nodes. The circuit elements in the equivalent network in Fig.1 are given as:

Impedances: $$Z_1 = j\omega\mu\Delta x \quad Z_2 = j\omega\mu\Delta y \; Z_3 = \frac{1}{(j\omega\varepsilon)\Delta x} \quad Z_4 = \frac{1}{(j\omega\varepsilon)\Delta y} \tag{6}$$

CCCS:

$$\begin{aligned} I_1 &= j\omega\mu\,\Delta y\left(V_{i+\frac{1}{2},j+1} - V_{i+\frac{1}{2},j}\right) \qquad I_2 = j\omega\mu\,\Delta x\left(V_{i,j+\frac{1}{2}} - V_{i+1,j+\frac{1}{2}}\right) \\ I'_1 &= j\omega\mu\,\Delta y\left(V_{i+\frac{3}{2},j+1} - V_{i+\frac{3}{2},j}\right) \qquad I'_2 = j\omega\mu\,\Delta x\left(V_{i+1,j+\frac{1}{2}} - V_{i+2,j+\frac{1}{2}}\right) \end{aligned} \tag{7}$$

Dependent Voltage Sources:

$$V_{TH} = \frac{J_{z,ext}(i,j)}{j\omega\varepsilon} \qquad V'_{TH} = \frac{J_{z,ext}(i+1,j)}{j\omega\varepsilon} \tag{8}$$

where, Δx and Δy are the grid spacing along the X- and Y- directions, respectively. Perfect electric conductor (PEC) and perfect magnetic conductor (PMC) boundary conditions are enforced by shorting and opening the nodal points along the boundaries of the simulation domain, respectively. This can be easily extended for the 3D case. For a lossless structure with N unknowns and p ports, this results in an equation of the form:

$$\left(s\mathbf{C} + \frac{\mathbf{\Gamma}}{s}\right)\mathbf{V}(s) = \mathbf{BI} \tag{9}$$

where, $\mathbf{C}, \mathbf{\Gamma} \in \mathbf{R}^{N\times p}$ are the nodal capacitance and susceptance matrices, respectively; $\mathbf{V} \in \mathbb{C}^N$, $\mathbf{I} \in \mathbf{R}^p$ are the nodal voltages and port excitation currents, respectively and $\mathbf{B} \in \mathbf{R}^{N \times p}$ is the incidence matrix for current excitation.

The imp-ENOR algorithm is then used to construct an orthonormal basis $\mathbf{Q} \in \mathrm{C}^{N \times q}$ and project (9) onto this basis, resulting in the reduced-order system of the same form

$$\left(s\tilde{\mathbf{C}} + \frac{\tilde{\mathbf{\Gamma}}}{s} \right) \tilde{\mathbf{V}}(s) = \tilde{\mathbf{B}}\mathbf{I} \tag{10}$$

where, $\tilde{\mathbf{C}} = \mathbf{Q}^T\mathbf{C}\mathbf{Q}$, $\tilde{\mathbf{\Gamma}} = \mathbf{Q}^T\mathbf{\Gamma}\mathbf{Q}$, $\tilde{\mathbf{V}} = \mathbf{Q}^T\mathbf{V}$ and $\tilde{\mathbf{B}} = \mathbf{Q}^T\mathbf{B}$. The above equations can then be solved very quickly to obtain the response over a wide range of frequencies.

III. Results and Discussion

To verify the accuracy of the equivalent-circuit based full-wave simulation followed by the improved ENOR approach, a power-ground structure, as shown in Fig. 2, is simulated. Two 15mm x 15mm thin metal planes connected by a via are considered, placed in a homogeneous dielectric medium (ε_r = 4.5) in a PEC box of dimensions 25mm x 25mm x 80µm. A unit cell of 0.5mm x 0.5mm x 10µm was used to discretize the structure shown in Fig. 2.

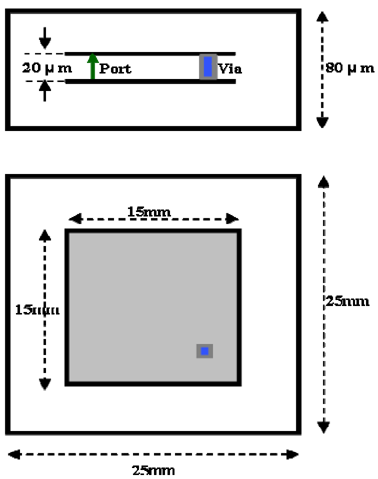


Fig.2 (a) Cross-section and (b) top-view of metal plane structure

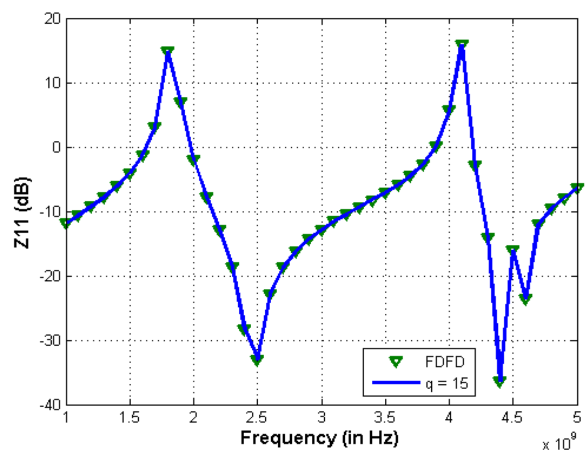


Fig.3 Z_{11} of the power-ground plane structure simulated using improved ENOR compared with full-wave 3D simulation [9]

The imp-ENOR algorithm is applied and the model-order is reduced to 15. Fig.3 shows a favorable comparison of the reduced model and the full-wave 3D simulation [9]. The average time per frequency point for the full 3D simulation was of the order of 120s, whereas the reduced-order system response took on the order of tenths of milliseconds per frequency point.

To demonstrate the convergence of the solution as the order of the reduced system is increased, a three-metal plane structure with aperture on the top plane is considered. The structure, with PEC boundaries and port placement is shown in Fig.4. The discretization of the 22mm x 22mm x 100µm volume is done using a unit cell of 0.5mm x 0.5mm x 10µm. The imp-ENOR algorithm is then applied, with the reduced order of the system varied as 2, 5 and 10. Fig.5 shows the impedance response of the two-port system for this set-up. As can be seen, as the reduced-order of the model is increased, the system response converges to the full-wave simulation. Whereas the 2-pole system is inadequate, the 5-pole system starts to diverge at frequencies above 4.5GHz. The 10-pole reduced order system shows a good match with the full-wave simulation results. The average time per frequency point for the full 3D simulation was of the order of 180s, whereas for the reduced-order system, it took on the order of tenths of milliseconds.

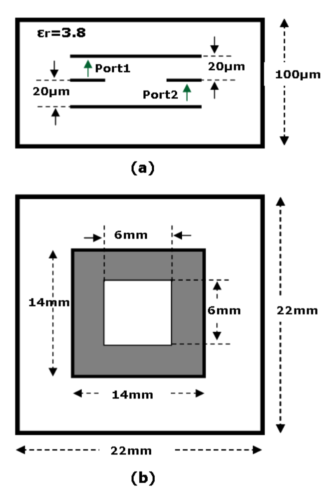
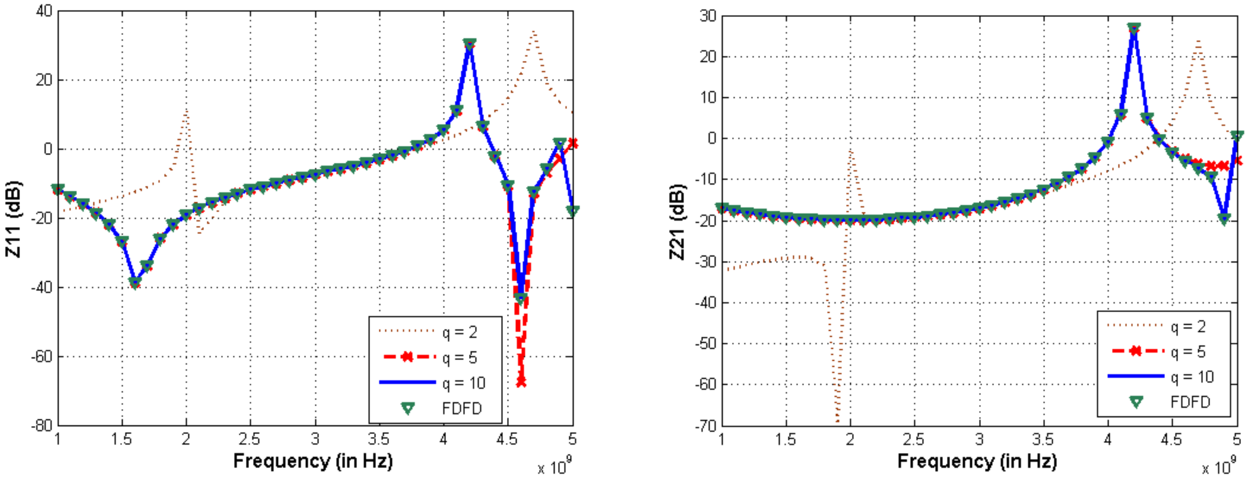


Fig.4 (a) Cross-section and (b) top-view of metal plane structure with aperture.



(a) (b)

Fig.5 Impedance parameters (a) Z11 and (b) Z21, for the metal plane aperture structure showing convergence as the order of the system is increased

In conclusion, the imp-ENOR approach has been applied to the 3D full-wave circuit-equivalent governing equation and the proof-of concept has been demonstrated by means of power-ground structure examples. The reduction in the order of the system from tens of thousands to double-digit values implies a significantly reduced simulation time. The inherent passive nature of the reduction ensures proper simulation without the need for additional passivity enforcement algorithms.

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