Memory Efficient Laguerre-FDTD Scheme for Dispersive Media

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Abstract—**The unconditionally stable Laguerre-FDTD method is suitable for simulating 3-D structures with large time step. In this work, memory efficient Laguerre-FDTD scheme for dispersive materials is proposed to ensure accurate modeling and less memory consumption compared to standard procedures. The memory efficient scheme is realized by representing the Laguerre domain expression of electric susceptibility in a recursive manner. Formulations have been derived for both Debye and Lorentz media. Numerical results show that the proposed Laguerre-FDTD method exhibits significant peak memory usage reduction and equivalent calculation accuracy of dispersive material involved transient simulation.**

Keywords-Laguerre-FDTD; memory efficient; dispersive material ; Debye and Lorentz media

I. Introduction

The finite-difference time-domain (FDTD) method has been widely used to solve transient electromagnetic problems for decades. To overcome the intrinsic stability issue due to the Courant-Friedrichs-Lewy (CFL) stability condition, semi-implicit and implicit FDTD schemes have been studied extensively. The alternating direction implicit FDTD (ADI-FDTD) method has been introduced which is shown to be unconditionally stable [1]. More recently, the locally-one dimensional FDTD (LOD-FDTD) method has been proposed with reduction of arithmetic operations and increased computational efficiency compared to ADI-FDTD method [2].

One of the major challenges in time domain methods is to rigorously and efficiently model the material dispersion. In [3], the frequency-dependent material is modeled by incorporating a discrete time domain convolution in conventional FDTD and is efficiently evaluated using recursion. However, this method still suffers from CFL condition which makes it computational inefficient to simulate multiscale structures. To ensure unconditional stability, the ADI-FDTD method has been extended to be able to simulate dispersive material in [4]. Also, the frequency-dependent implementation of LOD-FDTD has been reported in [2] which showed reduction of simulation time compared to explicit FDTD method.

In recent years, the unconditionally stable Laguerre-FDTD method has been proposed and extended with algorithm modifications [5]-[7]. By transforming the time domain problem to Laguerre domain using temporal Galerkin's testing procedure, the transient solution is independent of time discretization. Moreover, Laguerre-FDTD provides advantages in less numerical dispersion error when larger time step is used compared to ADI-FDTD. A Laguerre-FDTD formulation for frequency-dependent materials has been proposed in [7]. However, the formulation has a drawback of using all orders of solutions of Laguerre coefficients which requires considerable memory consumption to incorporate dielectric dispersion.

In this paper, memory efficient Laguerre-FDTD scheme for dispersive materials is proposed. To be specific, the electric susceptibility in time domain is obtained with Laguerre domain transformation which represents the Laguerre coefficient of susceptibility by the product of order-dependent and order-independent parts. By utilizing the unique mathematical properties of Debye and Lorentz models, the general Laguerre-FDTD formulations for dispersive materials are further rewritten into recursive form which significantly reduces the memory storage in the simulation.

This paper is organized in the following manner: In Section II, the memory efficient Laguerre-FDTD schemes for Debye and Lorentz media is introduced with formulations and derivation. In Section III, the proposed method is verified with numerical examples which show that structures with Debye and Lorentz media can be analyzed efficiently using Laguerre-FDTD method. In Section IV, we summarize some conclusions.

II. Proposed Scheme

A. General Formulations for Dispersive Materials

In Laguerre-FDTD method, time domain electric field components can be represented as a sum of infinite Laguerre basis functions $\varphi^q(\bar{t})$ scaled by Laguerre basis coefficient $\bar{E}^q$

$$\bar{E}(t) = \sum_{q=0}^{\infty} \bar{E}^q \varphi^q(\bar{t}) \tag{1}$$

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where $\bar{t} = t \cdot s$, s is the time scaling factor and t is time. Superscript q denotes the Laguerre coefficient of order q. The Laguerre basis functions $\varphi^q(\bar{t})$ can be expressed as

$$\varphi^q(\bar{t}) = e^{-\bar{t}/2} L^q(\bar{t}) \tag{2}$$

where $L^q(\bar{t})$ is the Laguerre polynomial which is defined recursively as

$$L^0(\bar{t}) = 1 \tag{3}$$

$$L^1(\bar{t}) = 1 - \bar{t} \tag{4}$$

$$qL^q(\bar{t}) = (2q - 1 - \bar{t})L^{q-1}(\bar{t}) - (q-1)L^{q-2}(\bar{t}),\ q \geq 2 \tag{5}$$

Assuming an isotropic, dispersive, lossy media, the wave equation can be written as

$$\nabla \times \nabla \times \bar{E} = -\mu \frac{\partial^2 \bar{D}}{\partial t^2} - \mu \frac{\partial(\bar{J} + \sigma \bar{E})}{\partial t} \tag{6}$$

where μ is the magnetic permeability, σ is the electric conductivity. $\bar{D}$ is the electric flux density which can be expressed as

$$\bar{D}(t) = \varepsilon_\infty \varepsilon_0 \bar{E}(t) + \varepsilon_0 \int_0^t \bar{E}(t-\tau)\chi(\tau)d\tau \tag{7}$$

where χ, ε_0 and ε_∞ are the electric susceptibility, electric permittivity of free space and infinite frequency relative permittivity, respectively.

Discretizing the differential equation (6) in Laguerre domain using temporal testing procedure yields

$$\begin{aligned} \nabla \times \nabla \times \bar{E}^q = &-\mu s^2 \left[\tfrac{1}{4}\bar{D}^q + \sum_{n=0,q>0}^{q-1} (q-n)\bar{D}^n \right] \\ &- \mu s \left[\tfrac{1}{2}\left(\bar{J}^q + \sigma \bar{E}^q\right) + \sum_{n=0,q>0}^{q-1} \left(\bar{J}^n + \sigma \bar{E}^n\right) \right] \end{aligned} \tag{8}$$

where the Laguerre coefficient of electric flux density is given by [7] as

$$\bar{D}^q = \varepsilon_\infty \varepsilon_0 \bar{E}^q + \varepsilon_0 \left(\sum_{n=0}^{q} \bar{E}^n \chi^{q-n} - \sum_{n=0}^{q-1} \bar{E}^n \chi^{q-1-n} \right) \tag{9}$$

Using (9), the frequency-dependent dispersion can be incorporated in the Laguerre-FDTD scheme. However, due to the form of the transformed convolution term, all previous solutions are required to calculate $\bar{D}^q$. Therefore, as the order of Laguerre coefficient increases, significant amount of memory is required to store all the solution of previous orders. Such large memory consumption is undesirable for solving practical 3-D problems.

B. *Debye Media*

The frequency-dependent Debye model of order n can be written as

$$\varepsilon(\omega) = \varepsilon_\infty + (\varepsilon_s - \varepsilon_\infty) \sum_{i=1}^{n} \frac{a_i}{1 + j\omega\tau_i} \tag{10}$$

where a_i and τ_i denote the strength and time constant of various relaxation processes, ε_s is the static permittivity. For simplicity, considering only the case for $n = 1$. The method can be easily extended to $n > 1$ cases in a similar manner. Thus, the frequency dependent susceptibility function is given by

$$\chi(\omega) = (\varepsilon_s - \varepsilon_\infty) \frac{a}{1 + j\omega\tau} \tag{11}$$

where a and τ are the strength and time constant of the first-order Debye relaxation process. Performing Fourier transform of (11), the time domain expression for susceptibility is

$$\chi(t) = \frac{a(\varepsilon_s - \varepsilon_\infty)}{\tau} e^{-\frac{t}{\tau}} \tag{12}$$

Using the definition in (1) and applied partial integration, with some manipulations, the Laguerre-domain expression for susceptibility is

$$\chi^n = \alpha_D \beta_D^2 \tag{13}$$

where

$$\alpha_D = \frac{2a(\varepsilon_s - \varepsilon_\infty)}{2 + s\tau} \tag{14}$$

$$\beta_D = \frac{2 - s\tau}{2 + s\tau} \tag{15}$$

Rewritten (9) into

$$\bar{D}^q = \varepsilon_\infty \varepsilon_0 \bar{E}^q + \varepsilon_0 \bar{G}^q \tag{16}$$

where

$$\bar{G}^q = \sum_{n=0}^{q} \bar{E}^n \chi^{q-n} - \sum_{n=0}^{q-1} \bar{E}^n \chi^{q-1-n} \tag{17}$$

and inserting (13) into (17) results in

$$\bar{G}^q = \alpha_D \left(\sum_{n=0}^{q} \bar{E}^n \beta^{q-n} - \sum_{n=0}^{q-1} \bar{E}^n \beta^{q-1-n} \right) \tag{18}$$

Subtracting $\beta_D \bar{G}^{q-1}$ from $\bar{G}^q$, we have

$$\begin{aligned} &\bar{G}^q - \beta_D \bar{G}^{q-1} \\ &= \alpha_D \left(\sum_{n=0}^{q} \bar{E}^n \beta_D^{q-n} - \sum_{n=0}^{q-1} \bar{E}^n \beta_D^{q-1-n} \right) \\ &\quad - \alpha_D \left(\sum_{n=0}^{q-1} \bar{E}^n \beta_D^{q-n} - \sum_{n=0}^{q-2} \bar{E}^n \beta_D^{q-1-n} \right) \\ &= \alpha_D \left(\bar{E}^q - \bar{E}^{q-1} \right) \end{aligned} \tag{19}$$

Therefore, $\bar{G}^q$ can be calculated recursively as

$$\bar{G}^q = \beta_D \bar{G}^{q-1} + \alpha_D \left(\bar{E}^q - \bar{E}^{q-1} \right) \tag{20}$$

For zero order

$$\bar{G}^0 = \alpha_D \bar{E}^0 \tag{21}$$

Inserting (20) and (21) into (16), the Laguerre coefficient for electric flux density $\bar{D}^q$ can be calculated recursively which requires only one previous order $(q-1)$ solution.

C. Lorentz Media

The frequency-dependent Lorentz model of order n can be written as

$$\varepsilon(\omega)=\varepsilon_\infty+(\varepsilon_s-\varepsilon_\infty)\sum_{i=1}^{n}\frac{a_i\omega_i}{\omega_i^2+2j\omega\delta_i-\omega^2} \quad (22)$$

where a_i, ω_i and τ_i represent the pole amplitude, the pole location and the damping factor. Again, only the case for $n=1$ is discussed here. The frequency dependent susceptibility function is given by

$$\chi(\omega)=(\varepsilon_s-\varepsilon_\infty)\frac{a\omega_0^2}{a\omega_0^2+2j\omega\delta-\omega^2} \quad (23)$$

The time domain expression of susceptibility can be obtained as

$$\chi(t)=\frac{a\omega_0^2(\varepsilon_s-\varepsilon_\infty)}{\sqrt{\omega_0^2-\delta^2}}e^{-\delta t}\sin\left(\sqrt{\omega_0^2-\delta^2}\right) \quad (24)$$

Introducing complex numbers, (24) can be rewritten into

$$\chi(t)=\mathrm{Im}\left[\frac{a\omega_0^2(\varepsilon_s-\varepsilon_\infty)}{\sqrt{\omega_0^2-\delta^2}}e^{-\left(\delta-j\sqrt{\omega_0^2-\delta^2}\right)t}\right] \quad (25)$$

Transforming (25) into Laguerre domain yields

$$\chi^n=\mathrm{Im}\left(\alpha_L\beta_L^n\right) \quad (26)$$

where

$$\alpha_L=\frac{a\omega_0^2(\varepsilon_s-\varepsilon_\infty)}{\sqrt{\omega_0^2-\delta^2}}\frac{2}{2\left(\delta-j\sqrt{\omega_0^2-\delta^2}\right)+s} \quad (27)$$

$$\beta_L=\frac{2\left(\delta-j\sqrt{\omega_0^2-\delta^2}\right)-s}{2\left(\delta-j\sqrt{\omega_0^2-\delta^2}\right)+s} \quad (28)$$

Performing the similar procedures as for Debye model, the electric flux density of the Lorentz model can be calculated recursively as

$$\bar{D}^q=\varepsilon_\infty\varepsilon_0\bar{E}^q+\varepsilon_0\mathrm{Im}\left(\bar{G}^q\right) \quad (29)$$

where

$$\bar{G}^q=\beta_L\bar{G}^{q-1}+\alpha_L\left(\bar{E}^q-\bar{E}^{q-1}\right) \quad (30)$$

and for zero order

$$\bar{G}^0=\alpha_L\bar{E}^0 \quad (31)$$

III. Numerical Results

To validate the memory usage improvement using the proposed scheme, a simple microstrip line shown in Fig. 1 is simulated and analyzed. The structure has a dielectric substrate with width and thickness of $s=30mm$ and $d=0.305mm$. The dielectric material is FR-4 and is assumed to be dispersive and can be approximated with first order Debye model. Parameters for Debye model are $\varepsilon_s=4.530$, $\varepsilon_\infty=4.398$, $a=1$, $\tau=57.22ps$ given in [8]. The metal strip is considered

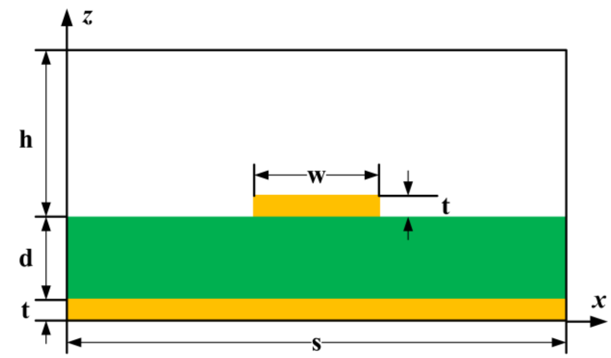

Fig. 1. Cross sectional view of the simulated microstrip line

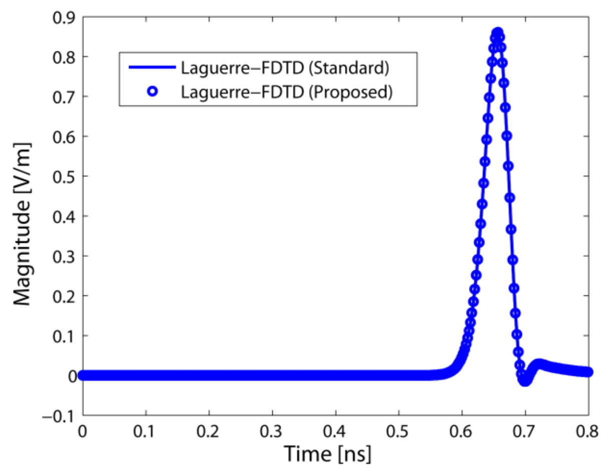

Fig. 2. Time domain response of the observation point of micrstrip line using (a) Standard Laguerre-FDTD (b) proposed memory efficient Laguerre-FDTD

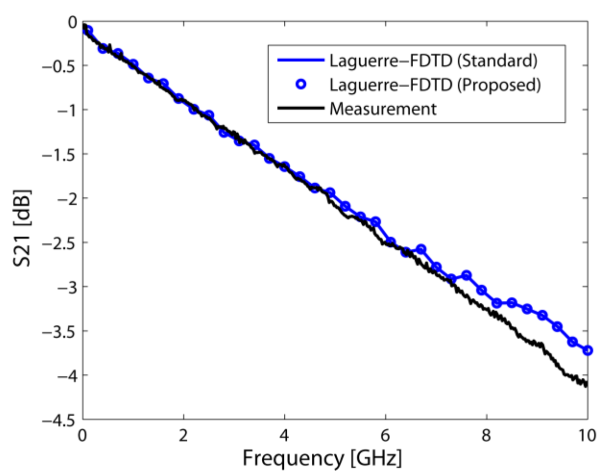

Fig. 3. Insertion loss of the simulated microstrip line with (a) Standard Laguerre-FDTD (b) proposed memory efficient Laguerre-FDTD and (c) measurement

as copper (conductivity $\sigma=5.8\times10^7 S/m$) with length, width and thickness of $l=93.5mm$, $w=0.51mm$ and $t=0.03mm$, respectively. The simulated structure is surrounded by an box

TABLE I
COMPARISON OF MEMORY CONSUMPTION FOR DIFFERENT SCHEMES

Test Case*	Method	Memory	Improvement
1	Standard Laguerre-FDTD	0.92GB	-
	Proposed Laguerre-FDTD	0.48GB	52.2%
2	Standard Laguerre-FDTD	1.85GB	-
	Proposed Laguerre-FDTD	0.95GB	51.4%

*Test case 1: Microstrip line; 2: Patch antenna

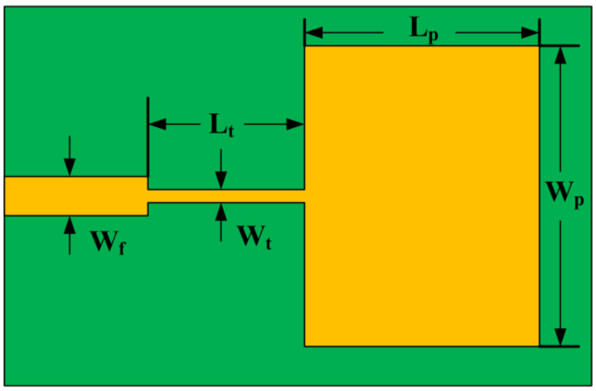

Fig. 4. Top view of the simulated microstrip patch antenna

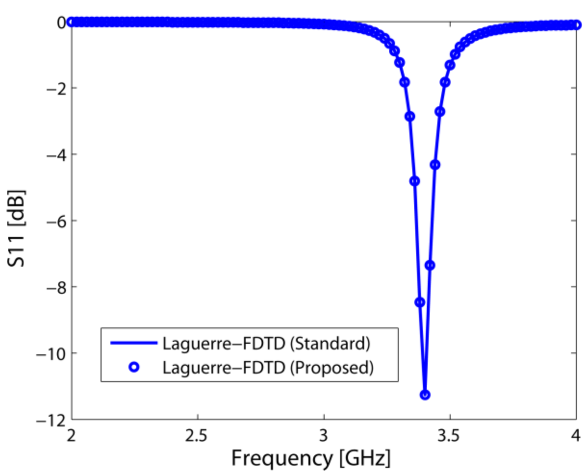

Fig. 5. Return loss of the simulated patch antenna with (a) Standard Laguerre-FDTD (b) proposed memory efficient Laguerre-FDTD

with first order ABC boundary with height $h = 1mm$. Two ports are set at each end of the metal strip.

Fig. 2 shows the time domain response of the observation point at one port of the microstrip line using the standard Laguerre-FDTD method and the proposed memory efficient Laguerre-FDTD method. Good agreement can be observed. Fig. 3 shows the comparison of insertion loss of the microstrip line using both methods along with the measurement. Good correlation with the measurement can be observed for both methods. The peak memory consumption using both methods are compared in Table I. It can be observed form Table I that under the same level of accuracy, the memory consumption for standard Laguerre-FDTD method is 0.92GB whereas the counterpart for memory efficient Laguerre-FDTD is 0.48GB. A 52.2% of improvement is achieved.

Fig. 4 shows the test case of a microstrip patch antenna. The structure has a dielectric substrate with thickness of $d = 0.8mm$. The dielectric material is FR-4 and is assumed to be dispersive. The Debye parameter of the material is the same as in the previous example. The metal strip is considered as copper whose conductivity is $\sigma = 5.8 \times 10^7 S/m$. The feature sizes shown in Fig. 4 are $W_f = 1.53mm$, $W_t = 0.4mm$, $W_p = 30mm$, $L_t = 12mm$, and $L_p = 20mm$, respectively. The simulated structure is surrounded by ABC boundary with one port assigned at the end of the feed line.

Fig. 5 shows the comparison of return loss of the patch antenna using the standard Laguerre-FDTD and the memory efficient Laguerre-FDTD. Good agreement can be observed. Table I shows the comparison of peak memory consumption for both methods. It can be observed that the memory consumption for standard Laguerre-FDTD and proposed Laguerre-FDTD are 1.85GB and 0.95GB, respectively. A 51.4% improvement is achieved.

IV. CONCLUSION

The memory efficient Laguerre-FDTD scheme for dispersive material is proposed. Formulations for both Debye and Lorentz media are derived in recursive manner. The proposed method is verified with results of structures with dispersive dielectric materials. Significant reduction of the peak memory consumption can be achieved using the proposed Laguerre-FDTD scheme.

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