2-D Non-Conformal Domain Decomposition Using the Laguerre-FDTD Scheme

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***Abstract*—A transient non-conformal domain decomposition scheme is proposed. The proposed scheme is based on the unconditionally stable Laguerre-FDTD method. A mortar-element-like method is applied and the field continuity at the non-conformal domain interface is enforced by using the Lagrange multiplier. The Schur complement method is used for addressing the interface problem. Simulation results have been presented to demonstrate the accuracy and efficiency of the proposed scheme.**

I. Introduction

The domain decomposition method has been studied for decades to solve electromagnetic problems. By dividing the computational domain into sub-domains, large problem can be decomposed and each sub-domain can be evaluated individually. In recent years, the non-conformal domain decomposition received wide attention since no conformality constrain is imposed at the domain interface [1]–[3]. Thus, mesh generation process can be significantly relaxed. One of the popular non-conformal approaches is the mortar element method, which enforces the field continuity between domains via Lagrange multiplier [1]. Another type of method, which is based on Robin transmission boundary condition, has also been successfully applied to large-scale electromagnetic problems, such as bandgap structure and antenna arrays [2], [3].

However, most of the implementation of the non-conformal domain decomposition methods are in the frequency domain. The time-domain schemes, such as finite-difference time-domain (FDTD) method, are not suitable in tackling the non-conformal domain interface because of the differential nature of the algorithm. Interpolation methods can be applied to address the field continuity but only with fixed grid ratio.

In this paper, time-domain non-conformal domain decomposition is realized using the Laguerre-FDTD method. Different from the conventional explict FDTD method, the Laguerre-FDTD method is implicit with unconditional stability [4]–[6]. Because of the subtle equivalency between the time-domain finite-element method (TD-FEM) and the Laguerre-FDTD method, a mortar-element-like method can be applied to address the interface continuity whereas the field equations inside each domain remain the same as in the standard Laguerre-FDTD scheme.

II. Proposed Method

Assuming an isotropic, non-dispersive, lossless media, the vector wave equation in time domain can be expressed as

$$\nabla \times \nabla \times \mathbf{E} + \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu \frac{\partial \mathbf{J}}{\partial t}. \tag{1}$$

Multiply (1) by an appropriate testing function $\mathbf{N}$, and integrate over domain results in

$$\begin{aligned} &\int_\Omega \left[(\nabla \times \mathbf{N}) \cdot (\nabla \times \mathbf{E}) + \mu\varepsilon \mathbf{N} \cdot \frac{\partial^2 \mathbf{E}}{\partial t^2} \right] dS \\ &= -\int_\Omega \mu \mathbf{N} \cdot \frac{\partial \mathbf{J}}{\partial t} dS. \end{aligned} \tag{2}$$

Considering the TE$_z$ case, in the computational domain, the electric field in x-direction is expanded with vector basis functions. After some manipulations with (2), the electric field coefficient equation in x-direction in Laguerre domain can be written as

$$\begin{aligned} &\left(\frac{1}{\bar{C}_y^E|_{i,j}} + \bar{C}_y^H|_{i,j} + \bar{C}_y^H|_{i,j-1} \right) E_x^q|_{i,j} \\ &+ \bar{C}_x^H|_{i,j}\, E_y^q|_{i+1,j} - \bar{C}_x^H|_{i,j}\, E_y^q|_{i,j} \\ &- \bar{C}_y^H|_{i,j+1}\, E_x^q|_{i,j+1} - \bar{C}_x^H|_{i,j-1}\, E_y^q|_{i+1,j-1} \\ &+ \bar{C}_x^H|_{i,j-1} E_y^q|_{i,j-1} - \bar{C}_y^H|_{i,j-1}\, E_x^q|_{i,j-1} \\ = &- \Delta\bar{y}_j J_x^q|_{i,j} - \frac{4}{\bar{C}_y^E|_{i,j}} \sum_{n=0,q>1}^{q-1} \sum_{m=0}^{n} E_x^m|_{i,j}. \end{aligned} \tag{3}$$

The electric coefficient equation in y-direction can be derived in a similar manner. Note that (3) is derived from TD-FEM with Laguerre discretization. This form is equivalent to the system equation obtained using finite-difference scheme in [4]. Therefore, a mortar-element-like of method can be applied to form interface coupling equation whereas the system equation representing the field inside each domain remains untouched.

For simplicity, considering the computational domain dividing into two sub-domains Ω_1 and Ω_2 with non-conformal domain interface as is shown in Fig. 1. By defining the Lagrange multiplier space as

$$\boldsymbol{\lambda} = \sum_{i=1}^{n} \boldsymbol{\varphi}_i \boldsymbol{\lambda}_i \tag{4}$$

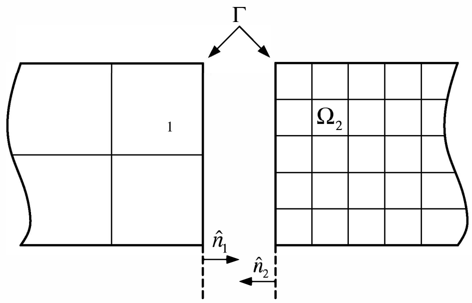

Fig. 1. Two domains with non-conformal domain interface in the Laguerre-FDTD scheme.

where n is the total number of expansion terms, $\boldsymbol{\lambda}_i$ is the unknown expansion coefficient, $\boldsymbol{\varphi}_i$ is the vector basis function, the following equations are obtained

$$\begin{aligned}\int_{\Omega_1}\left[(\nabla\times\mathbf{N}_1)\cdot(\nabla\times\mathbf{E}_1)+\mu\varepsilon\mathbf{N}_1\cdot\frac{\partial^2\mathbf{E}_1}{\partial t^2}\right]dS\\+\int_\Gamma\mathbf{N}_1\cdot\boldsymbol{\lambda}dl=-\int_{\Omega_1}\mu\mathbf{N}_1\cdot\frac{\partial\mathbf{J}_1}{\partial t}dS\end{aligned}\tag{5}$$

$$\begin{aligned}\int_{\Omega_2}\left[(\nabla\times\mathbf{N}_2)\cdot(\nabla\times\mathbf{E}_2)+\mu\varepsilon\mathbf{N}_2\cdot\frac{\partial^2\mathbf{E}_2}{\partial t^2}\right]dS\\-\int_\Gamma\mathbf{N}_2\cdot\boldsymbol{\lambda}dl=-\int_{\Omega_2}\mu\mathbf{N}_2\cdot\frac{\partial\mathbf{J}_2}{\partial t}dS\end{aligned}\tag{6}$$

$$\int_\Gamma(\mathbf{E}_1-\mathbf{E}_2)\cdot\varphi dS=0.\tag{7}$$

The resulting linear system for two domains together with the interface equation can be obtained as

$$\begin{bmatrix}\mathbf{K}_1 & \mathbf{0} & \mathbf{B}_1^T\\ \mathbf{0} & \mathbf{K}_2 & -\mathbf{B}_2^T\\ \mathbf{B}_1 & -\mathbf{B}_2 & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{E}_1^q\\ \mathbf{E}_2^q\\ \boldsymbol{\lambda}^q\end{bmatrix}=\begin{bmatrix}\mathbf{g}_1^q\\ \mathbf{g}_2^q\\ \mathbf{0}\end{bmatrix}.\tag{8}$$

By eliminating $\mathbf{E}_1$ and $\mathbf{E}_2$ in (8), we can derive the interface equation by Schur complement. Clearly, by introducing the Lagrange multiplier, the original 2-D problem is reduced to 1-D interface problem with fewer degrees of freedom. After the interface problem is solved, the electric field in each domain can be evaluated individually.

III. Numerical Results

To validate the proposed non-conformal domain decomposition method, the scattering from a PEC cylinder with square cross-section is investigated as is shown in Fig. 2. The wavelength corresponding to the upper frequency bound is chosen as $\lambda = 1\times10^{-2}$ m, and the side length of the square is $l = 2\times10^{-2}$ m. The calculation domain is decomposed into two sub-domains with interface grid ratio of 4:1. For comparison, conventional FDTD method and standard Laguerre-FDTD method are also used with the same meshing scheme as in domain one in the decomposed scheme. Fig. 3 shows the radar cross section (RCS). The CPU time for FDTD, Laguerre FDTD and domain decomposition methods are 50 s, 41 s, and 30 s, respectively. The domain decomposition scheme is accurate with the least CPU time.

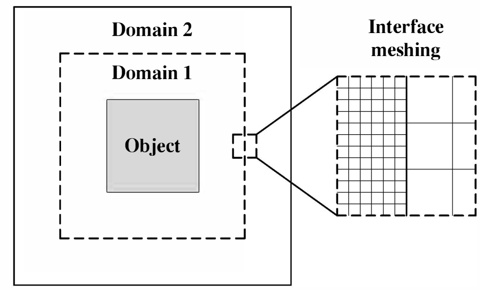

Fig. 2. Simulated square PEC cylinder structure with two domains.

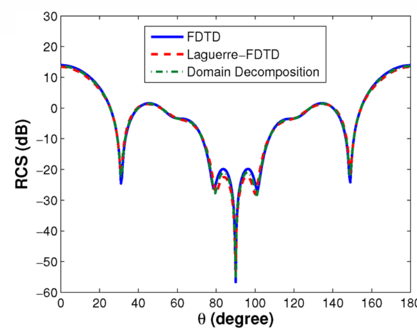

Fig. 3. Comparison of the calculated RCS with FDTD, Laguerre-FDTD and non-conformal domain decomposition.

IV. Conclusion

A non-conformal domain decomposition method in time domain has been proposed. By applying the Lagrange multiplier in Laguerre domain, the method is able to maintain the field continuity on the non-conformal domain interface. The numerical results show that the proposed method has good accuracy with improved computational efficiency compared to standard non-decomposed schemes.

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